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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

BALANCED DESIGN IN URBAN TRIANGULATION

By Charles D. Hopkins,[1] Esq.

Synopsis

It seems hardly necessary to stress the desirability for selective design since this first step in erecting engineering structures is decisive. Of course, the design, the mathematical processes, and the actual construction must be given equal weight if the operation is to be successful, consistent with the greatest degree of accuracy within the limits of justifiable expenditure. Granted the establishment of adequate mathematics and the development of competent physical implements (made available in the field of geodetics by the efforts of many practitioners through the years), it seems timely to make an effort to discuss the too-little-mentioned subject of design.

This paper, placing primary emphasis on the subject of triangulation design, nevertheless contains several statements regarding the mathematical processes. These statements are included solely to suggest some of the provocative ideas that were encountered during the course of the study, in the hope that further and more searching discussion may be stimulated.

Actual field results of applying the theories presented herein were so favorable as to be almost beyond understanding. In its beginnings, the idea of "balanced design in urban triangulation" originated from curiosity concerning the striking similarity in characteristics (of which there are many) of bridge and roof trusses and triangulation networks. For example, in the English truss a particular panel is used repeatedly, whose counterpart in triangulation is the component figure; but, because of physical limitations in the field, it would rarely be possible to erect a network of urban triangulation using only a single type of figure, such as the completed quadrilateral. Therefore, for very practical reasons, other types of figures are used which should be as closely related as possible.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by April 1, 1946.

[1] Geodetic Engr., Office, Chf. of Engrs., U. S. Army.

Chronologically, developments occurred in reverse order. The list of figure types discussed in Part 2 and designs for the field work mentioned in Part 3 were under way in 1937. In the interim, the field and final results (presented subsequently in Table 2) were being made available. As the results accumulated, the comparisons became apparent, and were deemed sufficiently unusual to warrant an attempt to find reasonable explanation for cause.

Introduction

An accurate triangulation scheme is the prime necessity as a foundation for the erection of a traverse structure to control property dimensions. This is particularly true in establishing geodetic control systems in urban areas since high property values prevalent therein are a most significant factor in determining the level of accuracy to be attained.

The desirable accuracy of horizontal urban control has been stated as not less than 1 part in 15,000 at the remote extensions of the traverse from triangulation.[2] This ratio of accuracy is based on the measurable precision within which a building can be constructed with relation to a given property line.

The control system, comparable to a local public utility to serve the needs of an area, should be capable of producing the required accuracy at any point within the jurisdictional limits where it may be demanded. Serious and unavoidable losses in accuracy must be anticipated in extending the control, from the triangulation through various phases of traverse, to define every property block in the area. It is desirable, therefore, that the basic triangulation be as strong as practical considerations will permit. Inherent weakness in the triangulation can cause almost insuperable difficulties, if the unpredictable distribution of discrepancies, in triangulation and traverse observations, is unfavorable and causes unacceptable localization of error. Localized error that cannot be corrected properly can rightly be called structural failure.

The necessity for well-designed figures at measured precision bases was recognized early, probably due to the fact that base lines were relatively short and the length element was undergoing rapid expansion to a longer average line of sight. The component figures of arcs of triangulation between measured bases, however, were not always so well designed, and in some cases simple triangles, singly or in groups, are found interspersed with much stronger geometrical figures. Similarly, local-area nets are sometimes characterized by mixed design. Absence of balanced design can produce only unpredictable results. For example, the triangulation nets in two adjacent cities, upon completion in the field, appeared equally acceptable; yet the final indexes of accuracy from the least-squares adjustments of each had little in common. One result was judged to be extremely good and the other very poor. Actually, both results were erratic accidents because selective predesign of the structures was not a controlling factor and consistency in design was lacking in both networks.

[2] "Technical Procedure for City Surveys," *Manual of Engineering Practice No. 10*, Am. Soc. C. E., 1934, pp. 74 and 106.

By nature, the triangulation structure is difficult to analyze, because it is an indeterminate structure, and, therefore, has no rigid explanation. It is an engineering structure, subject in general to the same physical laws, but differing in that, visibly, its physical make-up can be appreciated only on a detailed layout map. The scattered monuments that perpetuate the station locations on the ground do not afford any understanding of the scheme. It is possible, however, to identify certain characteristics of the structure by a study of the pattern of the distribution of discrepancies resulting from observations, and to arrive at rational conclusions that should be helpful.

Part 1.—Effects of Changes in Design

Obviously, any study of triangulation design based on actual arcs or nets would require enormous labor, and since many factors would modify final conclusions this idea was abandoned in planning the present study. By selecting types of actual component figures for individual examination, the outlook was made more promising, but even then certain difficulties remained, principally the lack of a reference base for common comparison. Although the absolute values of angles in actual triangulation can never be humanly known, discrepancies in observations can be distributed in the most consistent manner by a least-squares adjustment. Furthermore, changes in design would dictate the pattern of distribution. With these ideas in mind, the decision was reached to study the characteristics of design through comparison of the results from least-squares adjustments of ideal figures, whose absolutes are known.

For this purpose, four types of figures were selected—a central-point quadrilateral, a pentagon with and without central point, and a central-point hexagon. Using the perfect form for each figure wherein the absolute values of the angles are known, assumed discrepancies were applied to each angle. The assumed discrepancies were restricted in magnitude and range, and were uniformly applied—in tenths of seconds, 0.1 to 0.3, plus and minus successively, clockwise at each station and counterclockwise around the figure. By removing successive diagonal lines from the figure, the design could be changed by stages from saturation (that is, all diagonals being included) to the point where the figure becomes merely a combination of triangles. Thus, by making a series of adjustments for each stage of design, the various distributions of discrepancies could be tabulated and examined. In all, more than one hundred separate least-squares adjustments of the ideal figures were completed, and the results tabulated and graphed.

In the four types, the distribution of assumed discrepancies was made as similar as possible so that the final adjusted values would be closely comparable. Certain advantages are found in using the hexagon, however, since it covers wider range in design development and the results permit broader study and understanding. Therefore, the central-point hexagon is selected for discussion (see Fig. 1).

In making the series of least-squares adjustments for the hexagon (seven in all), the first involved the central-point figure without diagonals, and then the figure with the diagonals, included one by one in their numbered order.

Without diagonals the figure consists of six triangles in which the exterior lines lack double determination. With the inclusion of diagonal 1 (numbers

denoting diagonal lines are circled), the figure consists of a quadrilateral (in which all lines have double determination; that is, all lines are sides of at least two triangles) and four simple triangles. Diagonal 2 produces two quadrilaterals and two simple triangles; but, with the inclusion of diagonal 3, every line in the entire figure has reached double determination. In other words—the design-development stage being indicated by the number of included diagonals—at 0 the design is that of simple triangles, at 1 and 2 the design is mixed, and at 3 the design has reached the initial state of complete double determination. At 4 and 5 the design is classified as double determination increasingly compounded, and at 6 the ultimate of design is reached where double determination is completely compounded. It is pointed out that the inclusion of diagonal groups either 1, 2, and 3, or 4, 5, and 6 would produce double determination.

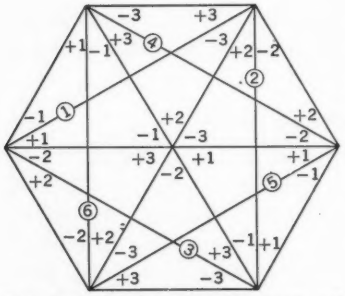


FIG. 1.—HEXAGONAL FIGURE, WITH THE PATTERN OF DISCREPANCIES GIVEN IN TENTHS OF A SECOND

The assumed discrepancies applied to the angles of the hexagon (Fig. 1), and the pattern and regularity of their distribution, produce a series of triangle closures that are of consequence to the study.

In any figure or combination of figures, there is a total in closures that is a constant. Referring to Fig. 1, it will be noted that the discrepancies at the perimetric angles within each of the radial triangles counteract each other; therefore, the triangle closure in each radial triangle will be identical to the discrepancy at its central-point angle. The algebraic sum of the central-point-angle discrepancies is zero, and thus the algebraic total-triangle closure in the entire figure is also zero. This total-triangle closure for the entire figure will remain constant regardless of the number of diagonals that may be inserted. The insertion of diagonals serves to divide and subdivide the discrepancies in the angles, and forms and reforms the resultant triangle closures. Likewise, the complementary side equations state and restate the proportional sine-to-side discrepancies. The total closure to be reconciled in each of the seven least-squares adjustments, therefore, remains a constant throughout the seven stages of design, upon which can be based a study of the mechanical changes produced in the series of adjustments, as evidenced by variations in the distribution of discrepancies.

After completing the series of adjustments, the corrected directions were tabulated. It then was possible, since the ideal hexagon is a perfect geometrical figure, to ascertain how efficiently the assumed discrepancies were distributed and what degree of approach to the absolute was attained. Therefore, another series of terms was tabulated—the absolute directions minus the corrected directions—which are designated Δ. The Δ-tabulations, without regard to sign, were then plotted, as shown in Fig. 2. On these graphs, the vertical section shows the increase in magnitude; and the horizontal section, the occurrence of Δ-terms in equal magnitude, to the total number of terms

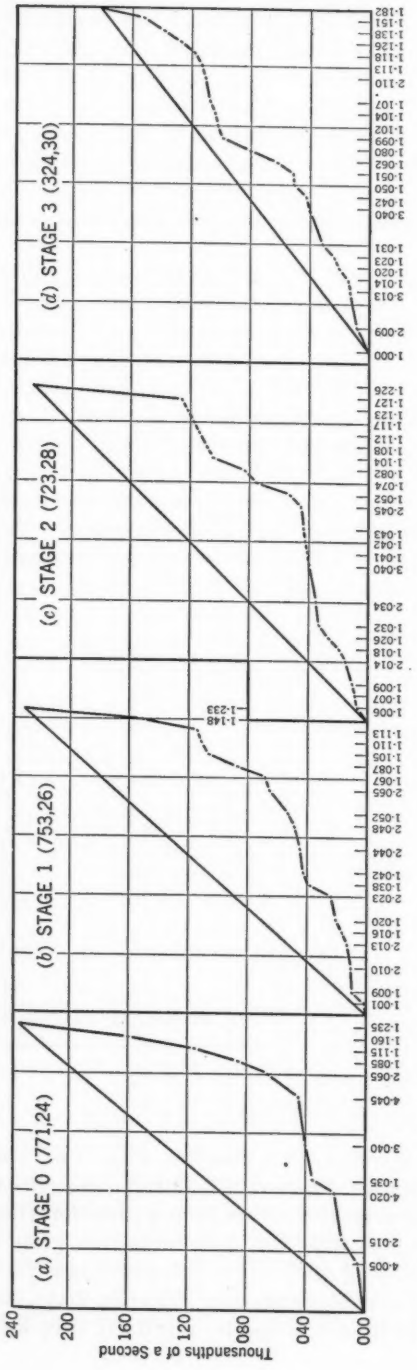


FIG. 2.—GRAPHS OF A-TABULATIONS FOR SEVEN STAGES OF DESIGN WITH INCREASING NUMBER OF DIAGONALS

involved. For example, in Fig. 2(*a*), the abscissa 1–235 denotes one Δ-term, and 235 denotes its Δ-value in thousandths; the abscissa 4–005 indicates that four Δ-terms of the same value have occurred. The meaning of the subcaption is explained as follows: In Fig. 2(*c*), for example, "stage 2" denotes a figure with two diagonals; "723" is the efficiency index; and "28" denotes the total number of Δ-terms. Subsequently, the writer will discuss these charts as comparative profiles depicting the state of geometry prevailing after the angle and side equations are satisfied—geometry upon which length equations are to be laid.

In order that a basis of comparison could be found to indicate a relative index of efficiency, an ordinate line was drawn connecting the extremes of the curve of the plotted Δ-terms. The ordinate areas were then scaled by planimeter and the resulting definitions of size indicated. Increased efficiency is thus approximated by the decrease in ordinate-area sizes. This unorthodox method of showing relative efficiency produced by changes in design was devised, and is believed to be a fair basis of comparison, because the inclusion of each successive diagonal introduces two more Δ-terms, and a realinement of all complementary Δ-terms is produced, with a tendency to approach progressively a straight-line distribution that will be more thoroughly compensated. This effect is very pronounced in the stages of Fig. 2, and would have ultimate influence on a length argument that passed through the figure.

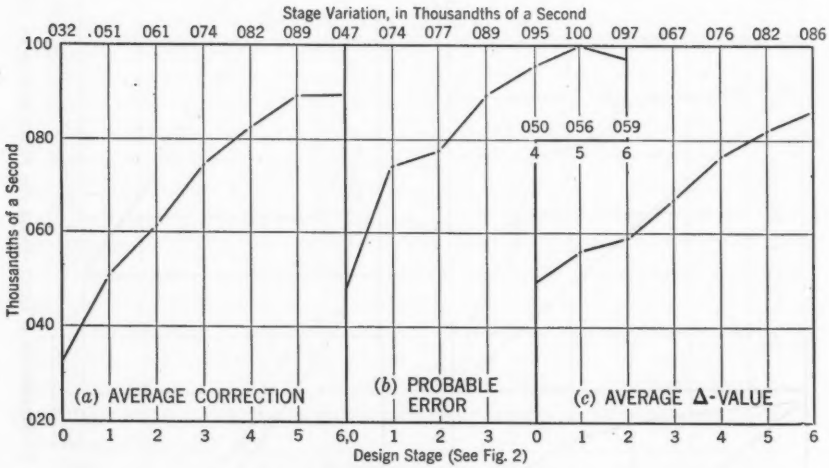

Stage Variation, in Thousandths of a Second
032 .051 061 074 082 089 047 074 077 089 095 100 097 067 076 082 086
050 056 059
4 5 6
100
080
060
040
020
Thousandths of a Second
(a) AVERAGE CORRECTION
(b) PROBABLE ERROR
(c) AVERAGE Δ-VALUE
0 1 2 3 4 5 6,0 1 2 3 0 1 2 3 4 5 6
Design Stage (See Fig. 2)

FIG. 3.—SUMMARY GRAPHS OF CRITERIA

Summary graphs of the average correction to a direction (Fig. 3(*a*)) and the probable error of an observed (assumed) direction (Fig. 3(*b*)), derived from the least-squares adjustments, and the average Δ-value from a direction (Fig. 3(*c*)) are inserted for use in conjunction with the Δ-ordinate-area graphs. These summary graphs are cross-referenced with the successive stages ((a), (b), etc.) in Fig. 2. For example, the ordinate-area index 771 (Fig. 2(*a*)), the average correction 0.032 (Fig. 3(*a*) stage 0), the probable error 0.047 (Fig. 3(*b*)

stage 0), and the average Δ-value 0.050 (Fig. 3(*c*) stage 0) can be considered collectively to complete the criteria for stage 0.

Certain conclusions can be stated on the information outlined on these graphs (Figs. 2 and 3). Because the triangulation structure is indeterminate, the conclusions cannot be facts, in the strict sense of that word, but it seems reasonable to believe that they can indicate strongly the trend of cause and effect. Incidentally, the specimen hexagon in its stages of design development can be considered as a single figure or as a small network of triangulation. In either case, the principle remains the same.

(*a*) Simple-triangle design and mixed design are not acceptable in high-precision triangulation, misleading final data from adjustment to the contrary. Stages 0, 1, and 2 (Figs. 2(*a*), 2(*b*), and 2(*c*)) plainly point to an unconcluded distribution of discrepancies. Stages 0, 1, and 2 in Figs. 3(*a*), 3(*b*), and 3(*c*) as related to stages 0, 1, and 2 (Figs. 2(*a*), 2(*b*), and 2(*c*)) show values that are misleadingly small. It is obvious that lack of geometrical conditioning, produced by diagonals, might result in either favorable or unfavorable distribution of discrepancies that could adversely affect a base-to-base chain, or any part of a network—a matter which will be referred to in Part 3.

(*b*) The severe decrease in the ordinate-area index in stage 3 (Fig. 2(*d*)) can be expected since double determination is completed. At this stage of design, a minimum of lines produces a maximum of results. The ordinate-area index, average correction, probable error, and average Δ-value have reached the lowest common minimum in double determination.

(*c*) Stages 4, 5, and 6 (Figs. 2(*e*), 2(*f*), and 2(*g*)) show a relatively smaller decrease in the ordinate-area indexes, but the related stages 4, 5, and 6 in Figs. 3(*a*), 3(*b*), and 3(*c*) show continued increase—with but two exceptions in stage 6—in average correction, probable error, and average Δ-value. Unless very exceptional circumstances warrant the inclusion of any or all of diagonals 4, 5, and 6, the resulting increase in time and cost is not justifiable; also, these diagonals are in excess of the uniform-figure design to be treated in Part 2.

(*d*) It is possible to summarize conclusions (*a*), (*b*), and (*c*) by reference to the theory of least squares and the exercise of logic. As applied to triangulation, the theory is given as follows: The sum of the squares of the corrections to the directions shall be a minimum. These terms for the design stages are: At stage 0, 0.0340; at stage 1, 0.1083; at stage 2, 0.1442; at stage 3, 0.2273; at stage 4, 0.2996; at stage 5, 0.3708; and at stage 6, 0.3919. If graphed, these terms would have the characteristic curve of increase shown in Fig. 3 for other final terms. The least-squares theory is satisfied independently in each of the seven design-stage adjustments, but it is necessary, further, to decide which least-squares result (that is, which stage of design) shall be chosen as the specific one that will probably be the most useful. At this point, the logic of economy should be the only deciding influence; and, as expressed in conclusion (*b*), the logic of economy points rather directly to the first state of complete double determination, produced by diagonals 1, 2, and 3, providing the maximum of results with the minimum of lines.

In planning and executing the foregoing investigation, the components are greatly reduced in magnitude for convenience in handling. Similarly, greatly

reduced models of trusses are used to study load stresses and subsequent reactions. Nevertheless, the process, however reduced, can guide thinking in connection with the much larger problem of planning and executing an actual urban network. The criteria for acceptance of field observations[3] constitute a restriction of the magnitude and range of discrepancies in observed angle values in somewhat the same manner that artificial restriction has been applied to the assumed discrepancies in the ideal hexagon. Of course, there is an infinite variety of patterns in the distribution of discrepancies since the structure is indeterminate, but the trends in results will be constant. Quite naturally, it should be remembered that the magnitude and range of actual discrepancies in a network are expected to be much larger and are correspondingly more decisive.

Part 2.—Uniform Unit Figures

To make the best use of advantageous trends, it is necessary to build up a reasonably complete series of closely related geometric figures, hereinafter to be called unit figures. They are designed to incorporate the useful characteristics suggested as regards the hexagon in Part 1. Such a series is presented in Table 1.

The ideal hexagon is the perfect form of the unit figure shown as type 4, Table 1. With shifts in the location of central points and changes in shape, the figures apparently become more difficult to identify; but this need not be misleading, since each type includes a definite number of physical members that is characteristic of one type only. In the series it will be seen that types 1 and 6 are used in varying combinations to make up the remaining composite types. All are equally useful in designing the urban net from reconnaissance operations.

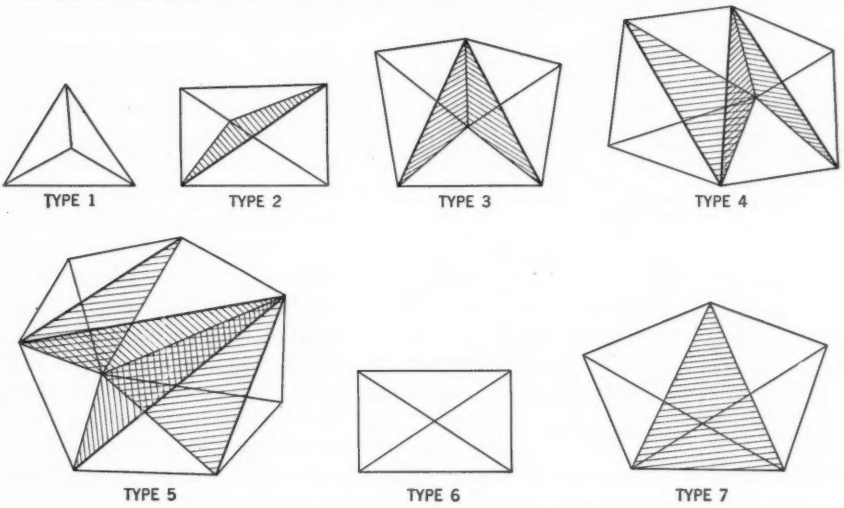
The types of unit figures shown in Table 1 are in two groups—one with central point and the other with diagonals only. The completed central-point triangle and the completed quadrilateral are the basic unit figures. In the first of these one triangle overlaps three and in the second two triangles overlap two (overlap in each case for double determination). In the composite types, attention is directed to the central-point figures and the regular increase in the number of diagonals employed; and in type 7, attention is directed to the pentagon with four diagonals. In that manner, types 2, 3, 4, and 5 combine a central-point triangle, in each, and one, two, three, and four quadrilaterals, respectively, in compound overlap (see crosshatched areas) needed for unit-figure geometry. Similarly, type 7 combines two quadrilaterals. The close relationship in visible characteristics is quite apparent in all these types. An indication of the close similarity in mathematical characteristics is shown by the reduced spread of the $\frac{D-C}{D}$ terms (see Table 1), in which:[4] D is the number of directions observed in the figure (always assuming that the starting line is held fixed); and C is the number of conditions to be satisfied in the least-squares adjustment.

[3] "Technical Procedure for City Surveys," *Manual of Engineering Practice No. 10*, Am. Soc. C. E., 1934, p. 11.

[4] *Ibid.*, Eq. 1, p. 14.

It will be noted that certain diagonals are not used; therefore, various choices in selecting the required number of diagonals are possible. This is a very useful circumstance in resolving reconnaissance and designing difficulties, and in the elimination of poorly shaped triangles. It is thus possible to design

TABLE 1.—SERIES OF COMPONENT UNIT FIGURES AND THEIR CHARACTERISTICS



Type	Unit figure (C-p = "central point")	Diagonals	$\frac{D-C}{D}$	Adjustment Conditions	
				Angle	Side
1	C-p triangle	....	0.60	3	1
2	C-p quadrilateral	1	0.56	5	2
3	C-p pentagon	2	0.55	7	3
4	C-p hexagon	3	0.54	9	4
5	C-p heptagon	4	0.53	11	5
6	Quadrilateral	2	0.60	3	1
7	Pentagon	4	0.56	5	2

the urban net by abutting component unit figures of relatively similar physical properties.

PART 3.—ACTUAL RESULTS IN THE FIELD

During the years 1937 to 1941, the writer collaborated with local personnel engaged in the construction of several urban triangulation nets. At that time, even though the idea of balanced design for triangulation existed only as a line of logic, based upon past experience and research, it was possible in most instances to demonstrate successfully the usefulness of the design, which was used subsequently in designing several urban networks. As an example for discussion the basic triangulation net for the City of Minneapolis, Minn., is reproduced in Fig. 4.

This network has sixteen component unit figures, including nine quadrilaterals, two central-point triangles, four central-point quadrilaterals, and one central-point pentagon. Two precision base lines were measured—Main-Columbia and Holy Angel-Cedar. The line Aggie-Snelling, established in length and azimuth by the U. S. Coast and Geodetic Survey, was held fixed, being related to the measured bases by condition equations in the least-squares adjustment; azimuth for the rigidly adjusted net was derived from the federal definition on this line.

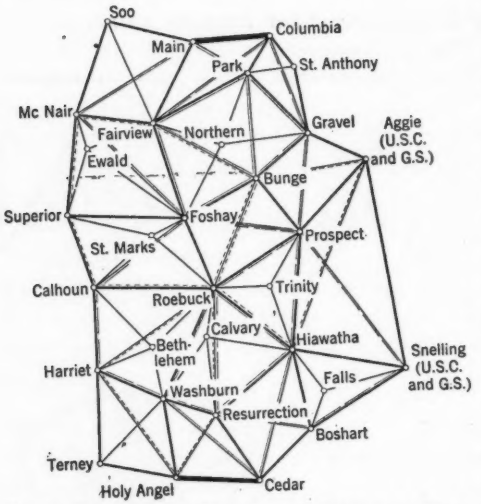


FIG. 4.—TRIANGULATION NET, CITY OF MINNEAPOLIS, MINN

As is apparent in Fig. 4, the first visible effect of balanced design is the ease with which the net and its component unit figures can be studied. In this connection, a very significant feature is the absence of lines, possibly intervisible, that would cross over the heavy lines accentuating each unit figure. Such lines would be superfluous and could only serve to complicate dangerously a more orderly design. The second effect, not so apparent but present nevertheless, is the pattern of geometrical condition equations necessary in the least-squares adjustment to distribute discrepancies from observations.

Disregarding the length argument for the moment, there are sixty angle equations and thirty-one side equations. The angle equations, all of which occur as unit-figure conditions, require no special comment. There are two distinct varieties of side equations, however: (*a*) Figure-side equations, twenty two in number, that occur complementally to the angle equations in the individual unit figures; and (*b*) vertex-side equations, nine in number, occurring at the interior stations that are the vertices of abutting lines between unit figures.

In this net the vertex stations are: Fairview, Park, Foshay, Bunge, Roebuck, Prospect, Washburn, Resurrection, and Hiawatha. The limit lines of the vertex equations are shown in Fig. 4 by lines of three styles placed on the side of the limit lines nearest the vertex station. These vertex-side equations serve to knit the scheme geometrically, by relating each unit figure to every other, as the figure-side and angle equations relate the members of each unit figure.

Specifically, the function of the vertex-side equation is to establish coincidence of the abutting lines between unit figures at the pole—that is, at the vertex station. Likewise, the figure-side equations produce coincidence of certain unit-figure lines at selected figure poles, and, in conjunction with the angle equations, complete the integration of the individual unit figures. The

process of integration proceeds from the individual unit figures, geometrically, through the vertex-side equations—relating figure to figure and vertex to vertex—to every part of the entire net. Each unit figure and its condition equations abut other unit figures and their conditions, and the vertex-side equations overlay each other and the unit-figure conditions in fish-scale fashion. There can be no doubt regarding complete integration and, furthermore, the highest possible degree of uniformity throughout the network.

Using transparent plastic sheeting, cutouts of the figure-side and vertex-side equations can be prepared; the selection of angle equations in each unit figure can be indicated on the layout map; and the cooperative effects, designed to produce mathematical rigidity, can be examined by overlay. In this way, also, the presence of an identical equation that would be fatal in the adjustment can be detected. Actually, this seems to be the graphic statics of triangulation whereby geometric integration becomes visible in form.

The length argument, superimposed as it is upon the geometrically conditioned network, can be considered separately. The necessary length equations, two in number, relate the measured bases to each other and to the fixed line through selected chains of triangles. There are discrepancies in passing from base to base and base to line, but, in view of the extreme accuracy of the measured lengths, these discrepancies after the satisfaction of the angle and side conditions can scarcely be considered as originating from the length measurements. Therefore, they represent the degree of failure in compensation in the angles used in the base-chain triangles, after these triangles have been effected by the angle and side equations, which are the geometrical conditions. Finally, the length equations (which are trigonometrical conditions) are superimposed and the total base-to-base discrepancy after adjustment is disclosed; and, if this final length discrepancy is highly acceptable, cannot it be considered as evidence that the geometrical adjustment, produced by the angle and side conditions, was but little disturbed by the length conditions? Furthermore, does it not afford evidence that all discrepancies were most consistently and favorably distributed, limited only by the mathematical process, throughout a network that is uniformly designed? In this connection, reference is made to the ultimate influence attributed to a compensated distribution of the geometrical discrepancies as suggested in Part 1.

Lastly, there are the final results from the least-squares adjustment that are accepted as the indicative proof. Criteria for the acceptance of urban triangulation schemes were presented in 1934 by the Committee of the Surveying and Mapping Division on City Surveys.[5] These specifications and the results of four actual operations, the networks of which were designed by abutting component unit figures of the types listed in Table 1, are summarized in Table 2.

In reviewing these somewhat surprising results, it should be remembered that, according to the theory of probability, any single result in this field can be called accidental, as can also any pair of results; but, when the third and the fourth are considered, the chance that the results are accidental becomes

[5] "Technical Procedure for City Surveys," *Manual of Engineering Practice No. 10*, Am. Soc. C. E., 1934, p. 12.

so remote that the circumstances call for another explanation—the inevitable conclusion in this case being cause and effect, with emphasis on balanced design as a basic cause.

To be sure, for all practical purposes, scheme C, Table 2, is just as useful as scheme B, despite the large difference in the adjusted base-to-base discrepancies. (The designations A, B, and C are used herein to identify three other city networks similar in design to the Minneapolis scheme.)

TABLE 2.—SUMMARY OF RESULTS, APPLIED TO URBAN TRIANGULATION

No.	Description	Manual 10[a]	Minneapolis[b]	Scheme A	Scheme B	Scheme C
	(*a*) RESULTS ATTAINED FROM FIELD OBSERVATIONS					
1	Triangle Closures, in Seconds: Average	1.50	0.91	1.22	1.31	1.09
2	Maximum	5.00	3.26	3.21	2.94	2.50
	Sine-to-Side Discrepancies, in Seconds:					
3	Average	0.70	0.56	0.48	0.64	0.49
4	Maximum	2.00	1.83	1.65	2.13	1.60
5	Probable error of measured bases[c]	600,000	1,657,000	1,638,000	1,678,000	1,850,000
6		.....	3,246,000	643,000	2,484,000	894,000
7		.....	(and one	1,862,000	3,853,000	978,000
8		.....	fixed fed-	2,051,000	....	....
9		.....	eral length)	2,234,000	....	....
	(*b*) FINAL RESULTS ATTAINED FROM THE LEAST-SQUARES ADJUSTMENT					
10	Base-to-base discrepancy[c]	100,000	1,478,000	2,782,000	3,378,000	378,500
11	Average correction to a direction, in seconds	0.75	0.61	0.64	0.54	0.49
12	Probable error of an observed direction, in seconds	0.75	0.75	0.54	0.64	0.59

[a] *Manual of Engineering Practice No. 10*, Am. Soc. C.E., 1934, p. 12. [b] Basic triangulation net for the City of Minneapolis, Minn. (see Fig. 4). [c] Ratios, 1 : 600,000, etc.

The significant fact is that, as a prerequisite to the other necessary phases of construction, it is possible by selective design to exceed the specifications for the length argument and to satisfy the other final criteria in four networks of similar design, without a single failure to date, at the same time producing the effect uniformly throughout each network. From networks of such consistent strength, extensions of traverse to provide adequately spaced control stations can be planned and executed very confidently, as has been amply demonstrated in practice.

It is disheartening to discover, during the development of a traverse layout from triangulation, the inexplicable presence of an area of weakness in an urban control system. It would seem to be only good judgment to build as strongly as is possible, rather than to be forced by circumstances to attempt the rationalization of disappointment that could have been avoided.

ACKNOWLEDGMENT

Fig. 4, the basic triangulation net for the City of Minneapolis, is reproduced with the kind permission of Frederick T. Paul, M. Am. Soc. C. E., City Engineer.

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PAPERS

SHIPWAYS WITH CELLULAR WALLS ON A MARL FOUNDATION

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Synopsis

Two full-length submerged shipways, whose walls consist of cellular cofferdams, are described in this paper. The cells of the cofferdams are filled with marl and founded on marl. The term "marl" is applied to a rather dense, greenish, calcareous sediment which has the grain-size characteristics of a fine, silty sand but, in an undistrubed state, the consistency of a stiff clay. The shipways are separated from each other by a partition wall which also consists of a cellular cofferdam. All the cells are of the diaphragm type. The hydrostatic uplift on the floors of the shipways is relieved not only by weep holes in the concrete mat but also, wherever necessary, by sand wells that drain the subsoil at a considerable depth below the floors.

The theoretical principles for the design of cellular cofferdams have been described elsewhere.[4] In connection with the design of the new shipways the local hydraulic and geologic conditions precluded an accurate forecast of the deflection of the walls and of the force that tends to lift the shipway floors. In this paper it will be shown that the risk due to such inevitable initial uncertainties can be eliminated completely by appropriate observations during construction, combined with modifications in the details of the design as soon as the required data become available.

General Layout and Soil Conditions

The general layout and two typical cross sections through the shipways are shown in Fig. 1. In a paper published in 1943, C. B. Jansen, M. Am. Soc. C.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by April 1, 1946.

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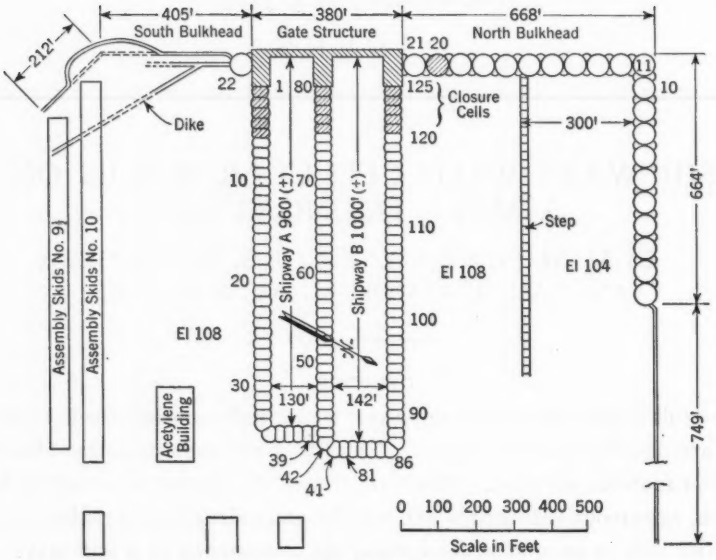
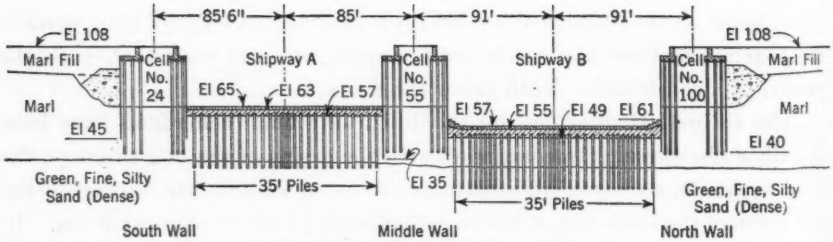
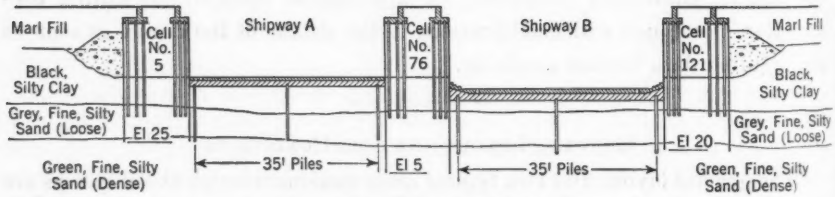
[4] "Stability and Stiffness of Cellular Cofferdams," by Karl Terzaghi, *Proceedings*, Am. Soc. C. E., September, 1944, p. 1015.



(*a*) GENERAL LAYOUT



(*b*) SECTION THROUGH EASTERN PART



(*c*) SECTION THROUGH WESTERN PART

FIG. 1.—PLAN AND TYPICAL SECTIONS OF SHIPWAYS

E., included a view of shipway B after completion.[5] While both shipways are empty the outside walls are acted upon by earth pressure whereas the middle wall carries no horizontal load. However, from time to time, one of the two shipways is flooded and pumped out again. Hence the middle wall must occasionally sustain full water pressure from one side. Since the crane-rail girders are located on top of the longitudinal sections of the walls it was highly desirable that the horizontal deflection of the crest of the walls after construction should not change by more than a few inches, or 1% of the height of the wall.

The soil conditions are indicated in Figs. 1(*b*) and 1(*c*). Prior to construction the site of the shipways was occupied by a shallow body of water whose depth increased from about 5 ft at the shore (eastern) side of the shipway to about 15 ft at the water front (west side). The site of the shipways and the adjoining areas were explored by eighteen 2-in. Shelby-tube borings. Before taking each sample the bottom of the hole was cleaned with a soil spoon whose diameter was slightly smaller than that of the casing. The samples were obtained in 2.5-ft sections and, in exceptional cases, in 5-ft sections. The sampler was forced into the clay by static pressure. After recovery the two ends of the Shelby tube were cleaned and sealed with a metal disk and paraffin. The samples were shipped in crates to the Soils Laboratory at Harvard University in Cambridge, Mass., where they remained in the Shelby tubes until they were tested. The routine tests included determination of the natural water content of every 6-in. section of every sample, and of the Atterberg limits and the unconfined compressive strengths of representative samples.

The tests showed that the uppermost stratum consisted of dark, silty, highly organic clay whose liquid limit ranges between 70 and 110. The natural water content is close to the liquid limit and the average unconfined compressive strength is about 0.2 kg per sq cm. North of the area occupied by the shipways the dark clay contains lenses of peat which suggest that this material was deposited during a low-water stage. The dark clay rests on a stratum of marl whose surface descends from about El. 90 at the east end of the shipways to about El. 25 or 30 at the west end. Within the area covered by the shipways the base of the marl stratum is fairly horizontal. Its elevation is between 15 and 30 ft. Hence, the thickness of the marl stratum decreases from about 60 ft or 70 ft at the east end to zero near the west end. Along its western rim the marl stratum is separated from the soft black top layer by a bed of fine, loose, silty sand. The marl rests on a thick bed of fine, dense, greenish sand with a variable admixture of silt.

The marl is a stiff, greenish soil which contains many sea-shell fragments. The grain-size curve of the marl is that of a fine silty sand. The coefficient of permeability is about 2×10^{-6} cm per sec which is that of a fine, silty sand with an effective size of about 0.02 mm. Slow triaxial compression tests furnished a value of 32° as the angle of internal friction of the marl, which is also the angle of friction for a silty sand. However, the Atterberg limits are those of a silty clay. The plastic limit is about 20 and the liquid limit varies between 30

[5] "Constructing the Shipways," by C. B. Jansen, *Civil Engineering*, July, 1943, p. 313.

and 40—in exceptional cases reaching 45. The natural water content ranges between 22 and 34. The apparent contradiction between the results of the mechanical analysis and the Atterberg limit tests suggests that the marl is completely flocculated. The clay and the fine silt particles are firmly united in hard clusters which behave like sand grains during the mechanical analysis. However, when the marl is remolded for the purpose of performing the Atterberg limit tests, the flocs are destroyed and, as a consequence, the character of the marl changes from that of a sand to that of a clay. This assumption is supported by the fact that remolding at an unaltered water content reduces the coefficient of permeability of the marl to about one thirtieth of its initial value. Remolding also reduces the shrinkage limit from about 20 to about 15 and it increases the compression index from about 0.05×10^{-5} to about 5×10^{-5}.

Design of Shipways

The walls of the shipways were designed on the assumption that the cells were to be filled with sand and gravel. However, immediately before beginning construction, the owners and the contractor agreed to fill the cells with marl because local experience had proved that artificial fills consisting of marl gradually become very stiff and stable. Since the base of the eastern and middle part of the shipway floors is located between 35 ft and 45 ft below the original surface of the marl, all the marl required for filling the cells became available while making the excavation for the shipways. Hence, the substitution of marl for sand and gravel involved a very considerable saving. On the other hand, it introduced an element of uncertainty into the design, which had to be compensated by careful observations in the field during construction and by occasional modifications of the design in accordance with the results of the observations.

The most vital decision in connection with the design of the shipway walls was in choosing the depth of penetration of the sheet piles. In a previous paper the writer has shown[4] that a cellular cofferdam on a soil foundation is not safe unless the bearing capacity of the sheet piles, per unit of width, is at least equal to 1.5 times the friction between the sheet piles and the fill of the cells, per unit of width of the sheet piles. When the shipways were designed, early in 1940, this rule was not yet known, and in the published theories for the design of cellular cofferdams, the depth of penetration received no consideration. Nevertheless, the contractor decided to drive all the sheet piles to firm bearing, regardless of the depth at which sufficient resistance is encountered. The sheet piles for some of the cells were driven to a depth of more than 50 ft. This precaution is largely responsible for the success of the job and it discloses remarkable practical judgment. To make sure that the sheet piles were well anchored and seated the sheet piles were driven at least to the depth at which the driving of test sheet piles required about twenty blows of a single-acting steam hammer (5,000-lb ram and 15,000 ft-lb per blow) per foot of penetration. In this connection it should be mentioned that a steel sheet pile which is interlocked with an adjacent pile offers a greater resistance against driving than a single one.

The design of the shipway floor was based on the observation that the floors of old dry docks in the vicinity of the new shipway did not heave. The floor of the deepest of these docks is at El. + 65, or 35 ft below mean tide level. These floors consist of lagging spiked to the caps of pile bents driven to support the ships. The floors of these dry docks have never shown any signs of heaving, which indicates that the upward hydrostatic pressure on the base of the marl stratum at about El. 30 (Fig. 1) at the site of these floors is smaller than the weight of the marl. If the floors of the old dry docks were covered, not with lagging, but with concrete, they could not be lifted except by water accumulating between the marl and the floors. Such an accident can be prevented easily by weep holes in the concrete floor. Hence, the designer duplicated the old floors, except that he replaced the lagging by a plain concrete mat punctured by weep holes consisting of 4-in. relief pipes. Prior to pouring the underwater concrete, the pipes were driven into the marl. These pipes are spaced 12 ft on centers on lines 4 ft out from each wall, and 22 ft on centers on lines about 48 ft apart across the shipways.[6] The water that comes out of the weep holes flows into covered drains and discharges into small pump sumps near the gate structure at the western end of the shipways.

Consideration of the geological situation at the site of the shipyards indicates that the conditions determining the danger of heave at the site of the new shipways are by no means necessarily identical with those at the site of the old dry docks. First, the distance between the center line of the middle wall of the new shipways and that of the deepest of the old dry docks is about 2,350 ft. Over such a distance the elevation of the base of the marl may vary considerably. Second, for a given thickness of the marl the hydrostatic uplift on the base of the marl stratum depends on the ratio between the permeability of the fine green sand and the marl. This ratio also varies from place to place within unknown limits. Third, part of the hydrostatic uplift on the base of the marl at the site of the old floors may be carried by shearing stresses along vertical planes through the outer boundry of the floors. Since the floor of shipway A is very much wider than that of either one of the floors of the older shipways, the perimeter shearing resistance per unit of area of the floor is smaller. This difference has an important influence on the hydrostatic pressure required to produce heave. Since the factor of safety of the old floors is unknown, all these circumstances had to be considered as potential sources of danger.

In connection with shipway B, the design of the floor on the basis of local precedent involved an additional risk because, during the period from the unwatering of the shipway to the placing of concrete to the finished floor level, the surface of the concrete floor was located at El. 55, or 10 ft below the level of the old floors. This situation was without any precedent at the site of the shipway floors. To make sure that the floor would not heave while the shipway was unwatered for the first time, the piezometric pressure in the pore water was measured at different points of the base of the marl stratum beneath the floor of shipway B at different stages of unwatering. The results of the observations are used to estimate in advance the hydrostatic pressure condition for complete

[6] "Constructing the Shipways," by C. B. Jansen, *Civil Engineering*, July, 1943, p. 310.

drawdown. If the pressure ever threatens to become excessive, it can be relieved by means of sand wells before the shipway is completely emptied.

In both shipways the concrete floor is supported by 35-ft green pine piles (or longer). The tops of all the piles were notched and encased in the concrete over a length of at least 2 ft. The layout of the piles was determined by the loads to be carried by the piles while ships are being built or repaired in the shipways. Each pile is assigned a maximum load of 20 tons.

The shipways are served by cranes that travel on tracks parallel to the center line of the shipways.[7] Each crane rail rests on the crest of a concrete wall which in turn is supported by piles.

The specifications required that the space behind the north and south walls of the shipways be filled to a distance of about 300 ft from each wall up to El. 108, and beyond 300 ft to El. 104. The western face of the backfill north of shipway B is supported by a cofferdam consisting of cylindrical cells with a diameter of 51 ft. This is known as the north bulkhead (Fig. 1). This bulkhead is remarkable inasmuch as it was constructed under exceptionally unfavorable soil conditions and no attempt was made to consolidate the marl in any of the cells except the closure cell. The results of the observations on the bulkhead have been published elsewhere.[4]

Incidents During Construction

The process of constructing the cofferdam and the construction equipment have been described by Mr. Jansen.[6] Within the area to be occupied by the shipway floors the marl was excavated by a heavy-duty dredging clamshell bucket operated from a derrick boat. It was either deposited directly in the cells or rehandled by barge and again by clamshell. In some instances the marl dropped out of the bucket in one big chunk, but usually it broke up into small lumps. During the first weeks after a cell was filled, the fill consisted of stiff chunks embedded in a matrix which was so soft that it offered very little resistance to the penetration of a man's finger. The sides, even of shallow test pits, had to be braced. The soil for filling the space behind the south wall was excavated by a suction dredge in a shallow borrow pit north of the shipway area and then was transported to the site through a pipe line about 2,000 ft long. In its natural state the material resembled the dark, silty clay which covered the marl at the site of the shipways. The filling operations started at the east end of the south wall. The fill consisted of clay balls with a diameter of about 4 in. embedded in a dark colored, almost liquid matrix. This material was so soft that it was not possible to walk on its surface.

During the construction of the cofferdams the deflection of the crest of the length walls (east-west walls) was ascertained about once every two weeks. While the highest part of the surface of the hydraulic fill was still 4 ft below final grade, the crest of the eastern part of the south wall adjoining the fill moved about 3 in. inward. During the following weeks, with the elevation of the top of the fill remaining unchanged, the greatest deflection of this wall in-

[7] "Constructing the Shipways," by C. B. Jansen, *Civil Engineering*, July, 1943, Fig. 2, p. 311.

creased to more than 5 in., as shown by the deflection curve a, Fig. 2. Subsequent driving of some of the piles for supporting the track for the gantry cranes south of the south wall further increased the greatest deflection to more than 7 in., as shown in curve b, Fig. 2, and the deflection continued to increase

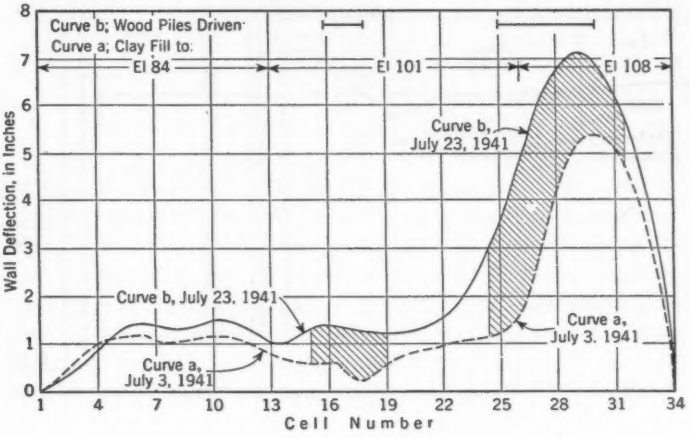


FIG. 2

at constant horizontal pressure on the wall at a considerable rate; yet the inside of the south wall was still acted upon by full water pressure. Since the removal of this pressure would triple the overturning moment it became increasingly doubtful whether the wall would survive unwatering the adjoining shipway.

The excessive deflection of the wall was obviously due to two independent causes—inadequate stiffness of the fill in the cells and the absence of internal friction in the hydraulic fill. To remedy the situation it was decided to increase the stiffness of the marl fill in the cells by compaction and drainage and to replace the hydraulic fill with gravel.

Replacement of the hydraulic fill by gravel offered an opportunity for a further reduction of the unbalanced pressure on the cofferdams by unwatering the gravel. Fig. 3, which is a vertical section through the gravel fill, shows that the gravel rests on marl and is covered with marl whose permeability is very low. After the east end of the gravel pocket also had been covered with marl, it was possible to maintain the water level in the gravel pocket adjoining the south wall, with a total length of almost 1,000 ft, at 20 ft below mean tide level by pumping not more than 0.5 gal per sec. This quantity is so small that it was later decided to divert the water from the gravel pockets, by pipes through the cellular walls, into the shipways and to pump it out of the sumps that collected the leakage from the weep holes and gate seats.

The effects of the partial replacement of the clay backfill by a gravel fill, and of draining the gravel pockets on the unbalanced horizontal pressure and on the overturning moment, are illustrated by Table 1. Line 1, Table 1, con-

tains the values corresponding to the original conditions, and lines 2 and 3 contain those corresponding to the modified design.

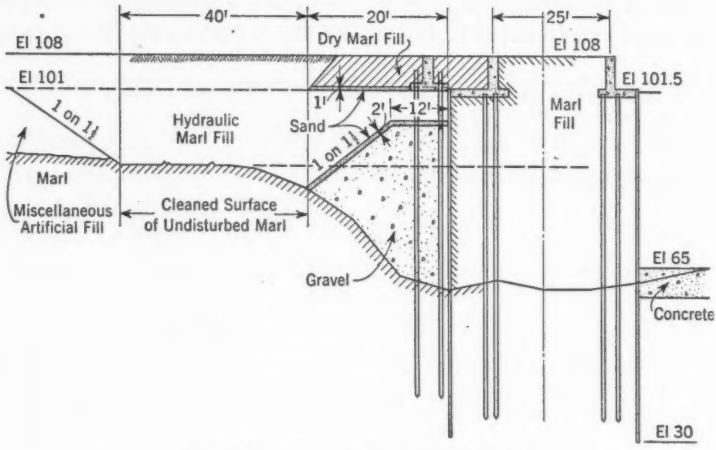


FIG. 3.—SECTION AT STATION 4 + 50 (±)

TABLE 1.—EFFECT OF DRAINAGE AND REPLACEMENT OF CLAY BACKFILL BY GRAVEL

Line	Description	HORIZONTAL PRESSURE (KIPS PER FOOT)		OVERTURNING MOMENT (KIP-FEET PER FOOT)	
		South wall	North wall	South wall	North wall
1	Wall entirely backfilled with black clay.....	112	....	1,605	....
	Modified Design:				
2	Water in the gravel pocket at El. 80.....	28.6	36.1	325	441
3	Gravel pocket flooded.....	53.7	65.8	669	905

The estimate of the values contained in Table 1 was based on the following data which were determined in the laboratory:

Material	Pounds per cubic foot
Brackish water.....	63.4
Saturated dark clay backfill.....	110
Submerged dark clay backfill.....	48
Saturated marl fill.....	120
Submerged marl fill.....	57
Moist gravel.....	112
Submerged gravel.....	67

The active earth pressure p_A on the wall was computed by means of Rankine's equation for cohesionless soils,

$$p_A = p \tan^2 \left(45 - \frac{\phi}{2}\right) = Np \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which: ϕ is the angle of internal friction; p is the vertical unit pressure in the soil at any depth; and $N = 1.00$ for the black clay backfill (original condition), and $N = 0.20$ for the gravel. To simplify the computation it was assumed that the top surface of the gravel pockets is located at El. 100 and not at El. 94, as shown in Fig. 3. Since the space between El. 94 and El. 100 was occupied by a marl fill subject to rapid consolidation between two horizontal, permeable layers, the error due to this simplifying assumption is unimportant. The marl backfill between El. 100 and El. 108 acts on the lower strata like a uniformly distributed surcharge of 960 lb per sq ft. The horizontal pressure exerted by the marl itself, above El. 100, has been disregarded because it is almost equal to zero. The active earth pressure of the marl in situ north of the north wall below El. 65 was disregarded because, in a natural state, the marl is very stiff.

Shipway A was unwatered in August, 1941, according to program. At that time the south wall was not yet backfilled and, as a consequence, it was acted upon by water pressure only. While the shipway was empty the deflection of the western part of the middle wall continued to increase very considerably at constant horizontal pressure on the wall. To stop the movement a temporary support was constructed as described subsequently herein.

While the water was being pumped out of shipway B, in January, 1942, the pressure-gage observations indicated exceptionally high piezometric levels at the base of the marl beneath the eastern part of the shipway. This pressure was relieved temporarily by installing shallow sand wells (lower end at about El. 20). At a later date, after the shipway had been in operation for more than a year, the provisional sand wells were replaced by permanent ones whose lower end is located at about El. 0.

Since January, 1942, observations have shown that the average movement of the crest of the middle wall due to the application of full water pressure alternately from the north and the south remains within a range of less than 3 in. which, from a practical point of view, is negligible. This result indicates that the attempt to consolidate the fill in the cells was successful.

Observations During and After Construction

The deflections of the crest of the walls were determined about once every two weeks by measuring the offset of each cell from a base line. One base line was established for every wall. Observations on the short north-south walls were begun but were soon discontinued because it became evident that the deflection of these walls is very small compared with that of the long east-west walls, for reasons discussed elsewhere.[4]

The deflection observations were supplemented by the measurement of the length of the upper edge of all the cross-walls of all the cells immediately after filling the cells with marl, after driving the wooden piles, and after driving the sand piles that were used for consolidating the fill in the cells. After shipway A was unwatered, the deformation of the inner walls of several cells of the adjoining cofferdams was surveyed carefully. The survey was repeated on a few cells of the middle wall after shipway B had also been pumped out.

Several months before shipway B was unwatered, seventeen pressure gages of the Bourdon type were installed in shipway A to obtain some information re-

garding the distribution of the pore-water pressure through the subsoil of the shipway; thirteen of these gages were connected with observation points located at a shallow depth below the shipway floor and four were connected with points located beneath the base of the marl stratum, at about El. 25. A short time before shipway B was unwatered, six gages were installed to measure the hydrostatic pressure at the base of the marl beneath shipway B and, in addition, four 6-in. observation wells were drilled from the crest of the north and the middle wall through the fill in the cells and the marl stratum into the fine, green sand. All the pressure gages were of the Bourdon type.

After construction was finished the deflection observations on the middle wall were repeated—(*a*) before one of the two adjoining shipways was flooded, (*b*) several times while it was flooded, and (*c*) several times after it was unwatered. The temporary pressure-gage installation was replaced by a permanent one, of thirty-one pressure gages, which furnishes fairly complete information on the pressure conditions at the base of the shipway floors. The well observations were discontinued, because the results cannot be relied upon unless the hydrostatic pressure conditions change very slowly. All the other observations will be continued until there is ample evidence that the elastic properties of the walls and the permeability of their base have become practically constant.

Sand-Pile Compaction of Fill in Cells

Since compaction of a saturated soil requires the expulsion of pore water, the efficacy of any attempt to compact a mass of saturated soil by mechanical means in a short time depends on the permeability of the soil. One of the most expedient means of compacting saturated relatively permeable soil consists in ramming sand or gravel into it. This method has been practiced for centuries in marsh lands. If the sand is rammed into holes made by driving piles or tubes into the ground, the columns of sand thus obtained are known as sand piles. The process of ramming increases, temporarily, the pressure in the soil surrounding the seat of the successive impacts and the excess water can escape in a horizontal direction into the voids of the sand piles. By bailing or pumping from the sand piles for several weeks after their installation, simple gravity drainage is added to the effect of the drainage produced by driving the sand piles.

If the permeability of a soil is as low as that of a fine silt or clay, driving sand piles into it merely causes the soil to rise between the sand piles with almost no change in water content and the process of subsequent gravity drainage is too slow to be of practical value. A failure to consolidate a clay fill in cells by gravity drainage was experienced on the cofferdam used for raising the *Maine*. The fill consisted of sandy to heavy clay excavated by means of a hydraulic dredge. In some cells the water was pumped from 3-ft by 3-ft shafts and in others from 8-in. wells with perforated walls;[8,9] yet the effect of pumping was disappointing. In the process of drainage toward wells

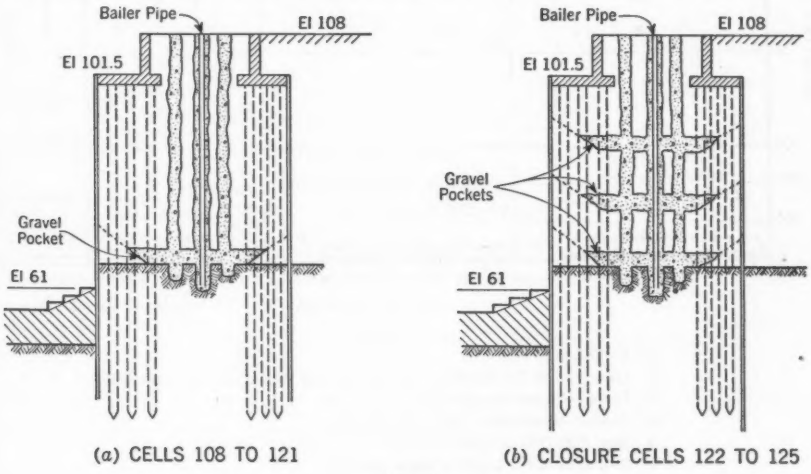
[8] *Engineering Record*, Vol. 63, 1911, pp. 548–549.

[9] Discussion by D. A. Watt of "Unusual Coffer-Dam for 1,000-Foot Pier, New York City," by Charles W. Staniford, *Transactions*, Am. Soc. C. E., Vol. LXXXI (December, 1917), p. 544.

the hydraulic gradient is maintained very much longer than during the process of driving sand piles into the soil. Hence, if bailing from filter wells fails to produce noticeable consolidation, it is practically certain that the driving of sand piles would also be ineffective.

The fill in the cells of the shipways consisted of stiff chunks of marl embedded in a soft matrix. The sole purpose of compaction was to weld the stiff chunks together by driving the excess water out of the matrix. Owing to the peculiar properties of the marl, the matrix seemed permeable enough to permit fairly rapid drainage. Therefore, it was decided to try the sand-pile procedure, and experience showed that the decision was justified.

Every sand pile is inevitably surrounded by a mantle of remolded marl whose permeability is very much smaller than that for the remainder of the fill. In order to establish, within the fill, a large unobstructed area of contact between fill and drain in addition to the partly obstructed area of contact with the sand piles the design illustrated in Fig. 4 was finally adopted. Fig. 4 (*a*) is



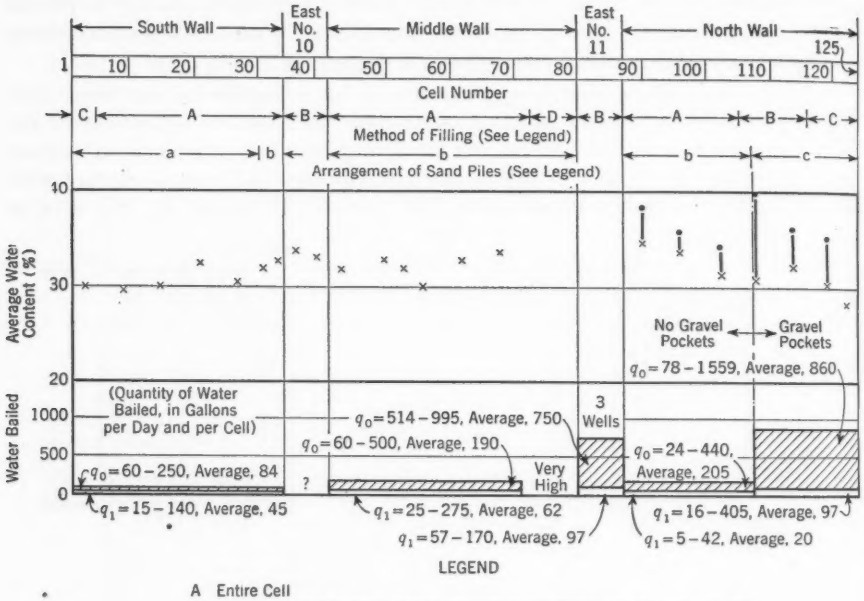
(*a*) CELLS 108 TO 121 (*b*) CLOSURE CELLS 122 TO 125

FIG. 4

a vertical section through the cells of the middle part of the north wall and Fig. 4(*b*) is a section through the closure cells at the west end of this wall. Each cell contains a central bailing well and one (Fig. 4(*a*)) or three (Fig. 4(*b*)) gravel pockets. Prior to placing the gravel for the pockets, marl was dumped into the cell along the inner and outer sheet-pile wall to intercept free communication between the gravel pockets and the open seams in the locks of the sheet-pile walls. During the filling operations the site for the bailing well was occupied by a 12-in. casing whose lower end rested on the surface of the lowest layer of gravel. After the cell had been filled with marl, a 4-in. bailer pipe was lowered into the 12-in. casing; the space between the two casings was filled with gravel; and the outer casing was pulled. The next step consisted in driving the sand piles, as described elsewhere by Mr. Jansen.[6] The driving of additional sand piles was discontinued when the total number of blows required to drive the shells for the piles (12-in. tubes with capped lower end) be-

came equal to 5,000. For the north wall, whose cells contained a softer fill than the other walls, 6,000 blows were specified.

Immediately after a sand pile was installed the voids in the sand were empty. However, as soon as the next pile was driven, the sand pile became saturated, and the water flowed out of the sand and flooded the surface of the fill. This phenomenon demonstrated the relatively high permeability of the matrix in the fill. After the first sand pile was installed, bailing from the well



LEGEND

A Entire Cell
B Lower Half of Cell Filled Directly from Shipway; Remainder from Borrow Pit
C Entire Fill Taken from Borrow Pits
D Random Mixture, 50% Marl+50% Gravel
a Sand Piles Only in Outer Half of Fill
b Central Sand Pile Cluster+Single Sand Piles
c Sand Piles+Gravel Pockets as Shown in Fig. 4
• Water Content of Marl After Deposition
× Water Content of Marl After Consolidation

FIG. 5

was begun. Records were kept of the quantity of water removed from each cell per day. They showed that the inflow into each well decreased at a decreasing rate. After about three weeks the inflow became constant, which indicated that the water content of the fill ceased to decrease. Hence, about ten days later bailing from this cell was discontinued. Information regarding the initial and the final rate of flow toward the wells is given at the bottom of Fig. 5.

A constant flow toward the wells can be maintained only by seepage from permanent sources of water into the cells, partly through the marl stratum which constitutes the base of the fill in the cells and partly through the locks of the sheet piles. Approximate computations based on the flow-net method

indicated that the observed rate of flow and the measured hydraulic gradients can be accounted for only by assuming that the loss of head associated with the percolation of the water across the rows of sheet piles was negligible.[4]

In order to collect information on the effect of the sand piles on the water content of the marl fill in the cells, 22 Shelby-tube borings were made from the top of the dam to the base of the fill in the cells. From each hole a continuous

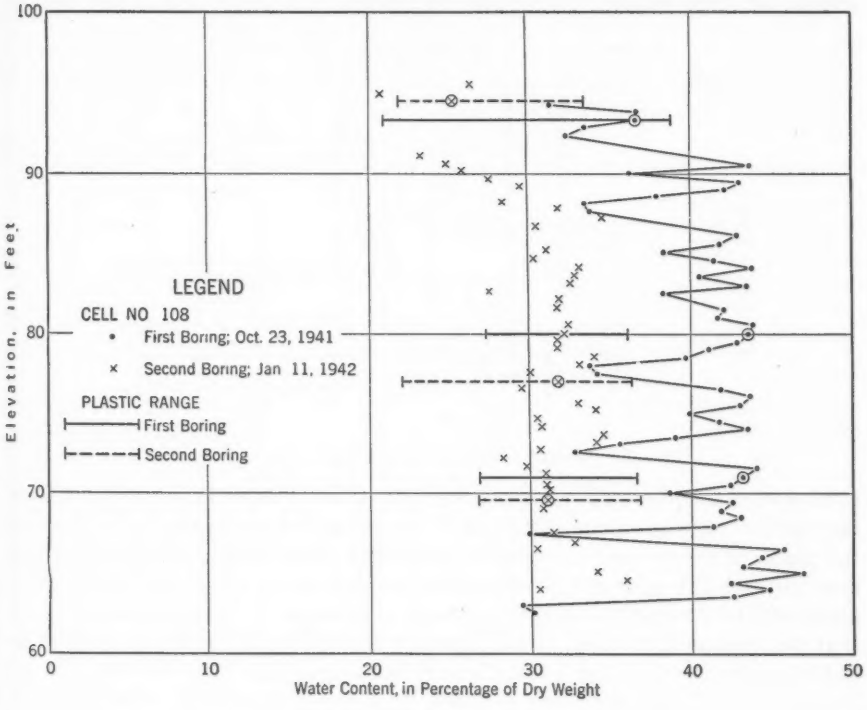


FIG. 6

set of 2-in. samples, 2.5 ft long, was recovered. In the cells of the north wall the borings were made in duplicate, one immediately after the cells were filled and one after bailing was discontinued. Fig. 6 shows the results of the water content and Atterberg limit determination for cell No. 108 in the north wall. Before the sand piles were installed the water content of many samples was high above the liquid limit. After consolidation the average water content was roughly equal to that of the parent material in its original state. Fig. 5 represents a digest of the results of the tests and observations.

After all the cells were filled and drained, test shafts were excavated in six cells of the south, middle, and north walls from El. 72 to El. 75. In all these shafts the marl appeared to be very firm and the matrix (which, at the outset, was a source of justified concern) had become as firm as the undisturbed marl.

Hydrostatic Pressure Conditions Beneath The Shipway Floors

Fig. 7 is a simplified vertical section through the middle wall and the adjoining shipways A and B. The shipway floors are tied to the foundation piles and the sum of the resistance of the individual piles against pulling is considerably greater than the weight of the soil between the piles. Therefore, the floors cannot be lifted without lifting, simultaneously, the major part of the body of

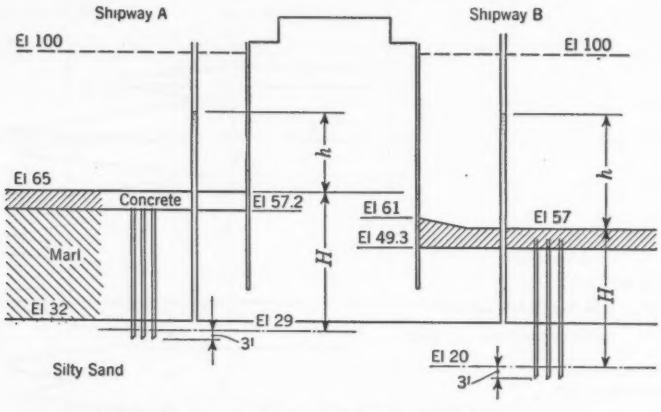


Fig. 7.—Simplified Section Through the Middle Wall

soil between the piles. The lower ends of the piles are very thin and can readily be pulled out of the surrounding soil. Therefore, it is assumed that the potential plane of separation between the stationary mass of soil and the mass that goes up with the piles is 3 ft above the level of the points of the piles. This plane will be referred to as the "plane of separation." It is further assumed that the piezometric levels at the plane of separation are identical with those at the points at which the water pressure is being measured. These points are located at some distance above or below the plane of separation. Nevertheless the preceding assumption is justified because the observations on the shallow gages have shown that the hydraulic gradient in a vertical direction is very small.

In the following p_t equals total submerged weight of soil, concrete, and water located between the plane of separation and the surface of the shipway floor (total weight reduced by full hydrostatic uplift), in pounds per square foot of the shipway floor; h equals hydrostatic head with reference to shipway floor (difference between the elevation of the piezometric level and the elevation of the shipway floor); H equals vertical distance between the plane of separation and the shipway floor; and γ_w equals unit weight of water. The force that tends to lift the submerged mass of soil and concrete above the plane of separation is $h\,\gamma_w$ and the force that resists this tendency is p_t per unit of area. Therefore, the factor of safety with respect to a rise of the floor is equal to the ratio

$$G_s = \frac{p_t}{h\,\gamma_w} \qquad (2a)$$

provided the shipway is empty. If the water rises in the shipway to an elevation h_1 above the floor, the resisting force remains unchanged while the uplift decreases from $h\,\gamma_w$ to $(h - h_1)\,\gamma_w$. Hence,

$$G_s = \frac{p_t}{(h - h_1)\,\gamma_w} \qquad (2b)$$

When computing the resisting unit force p_t it was assumed that the unit weight of the concrete is equal to 156 lb per cu ft and that of the saturated marl 110 lb per cu ft. Since it was found that the water escaping from the weep holes and sand wells is fresh, it was assumed that the unit weight of the water is equal to 62.4 lb per cu ft. The numerical values of the elevations of the boundaries between different materials are given in Fig. 7.

In shipway A the points of the piles are at El. 26, the plane of separation at El. 29, the base of the concrete at El. 57.2, and the surface of the concrete at El. 65; whence, $p_t = 28.2 \times 110 + 7.8 \times 156 - 36 \times 62.4 = 2{,}080$ lb per sq ft.

In shipway B the points of the piles are at El. 17, the plane of separation at El. 20, the base of the concrete at El. 49.3, and the surface of the concrete at El. 57; whence, $p_t = 29.3 \times 110 + 7.7 \times 156 - 37 \times 62.4 = 2{,}110$ lb per sq ft.

In Fig. 8, curve C_a represents the relation between the elevation of the piezometric level and the corresponding factor of safety G_s for the floor of the empty shipway A, and C_b represents the corresponding relation for the empty shipway B. By means of these curves one can determine, for any given piezometric elevation, and without any computation, the corresponding factor of safety G_s of the shipway floor, provided the shipway is empty. If the water stands in a shipway at an elevation h_1 above the floor, the corresponding G_s can be computed by means of Eq. 2*b*.

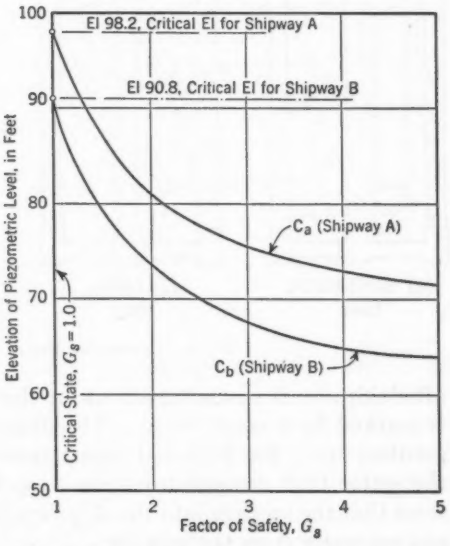


FIG. 8.—RELATION BETWEEN THE ELEVATION OF THE PIEZOMETER LEVEL AND THE FACTOR OF SAFETY

Fig. 8 shows that the danger of a heave of the floor of shipway A does not arise under any conditions of operation. However, for shipway B both the theoretical data given in Fig. 8 and the observations during the process of unwatering shipway B in January, 1942, indicated that the ultimate factor of safety with respect to the heave of the floor at the east end of shipway B would be considerably smaller than 1.5. To increase the factor of safety for this section of the floor, twenty-eight sand wells were installed during the process of unwatering in the following manner: 3-in. pipes were driven through 4-in.

openings in the eastern part of the concrete floor of shipway B and through the marl into the greenish sand to about El. 20. Then the interior of the pipe was emptied by means of an air jet and the pipe was pulled. While pulling the pipe its lower end was always kept full of coarse sand. Thus a vertical drain was formed with its lower end at about El. 20.

After shipway B was completely pumped out the temporary system of pressure gages was replaced by a permanent one comprising thirty-one gages. The observation points were beneath the base of the marl, at about El. 25. Fig. 9(*a*) shows the curves of equal piezometric elevation, about six months

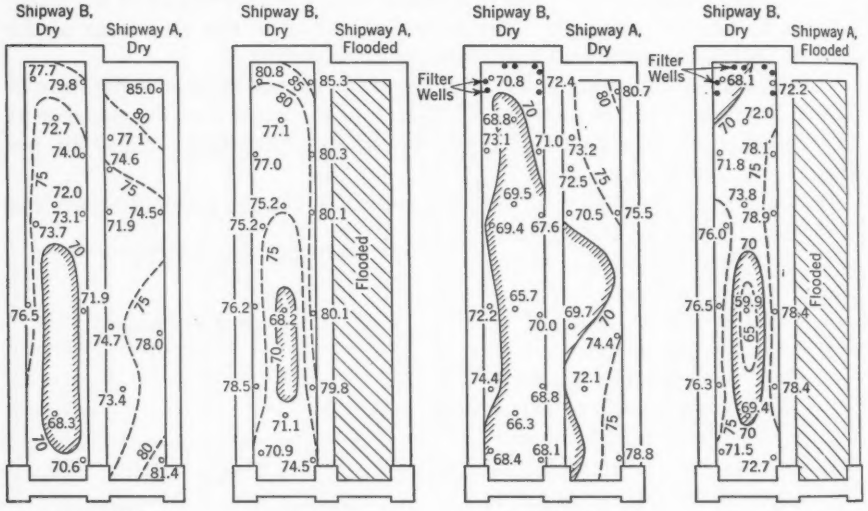


FIG. 9.—CURVES OF EQUAL PIEZOMETRIC ELEVATION

after shipway B was pumped out for the first time. The location of each gage is marked by a small circle. The diagram indicates a fairly steep hydraulic gradient from the land end (east) toward the outboard end. Furthermore, the water that escaped from the weep holes was fresh. These observations show that the seepage into the shipways is ground water from the land side and not sea water from the estuary.

The total leakage through the weep holes (630 gal per hr) was several times greater than the quantity computed on the assumption that the water flows toward the concrete floor only through the marl. This finding indicates that a considerable part of the leakage follows the rows of sheet piles in a vertical upward direction.

As often as a shipway is filled with water, the hydrostatic pressure at the base of the marl stratum goes up, but subsequent unwatering reduces it again to the initial value. The time lag between the change of the water level in the shipway and the corresponding change of the pore-water pressure at the base of the marl beneath the adjoining shipway is very small. It can be accounted

for by a slight swelling of the marl due to the increase of the seepage pressure at unaltered total pressure during the filling of the shipway and subsequent slight reconsolidation while the shipway is again being pumped out. Fig. 10 shows the effect of flooding and unwatering shipway B on the readings of two of the gages in shipway A and on the water level in an observation well in the north wall. The abscissas represent the time; the ordinates of curve (1) indicate the

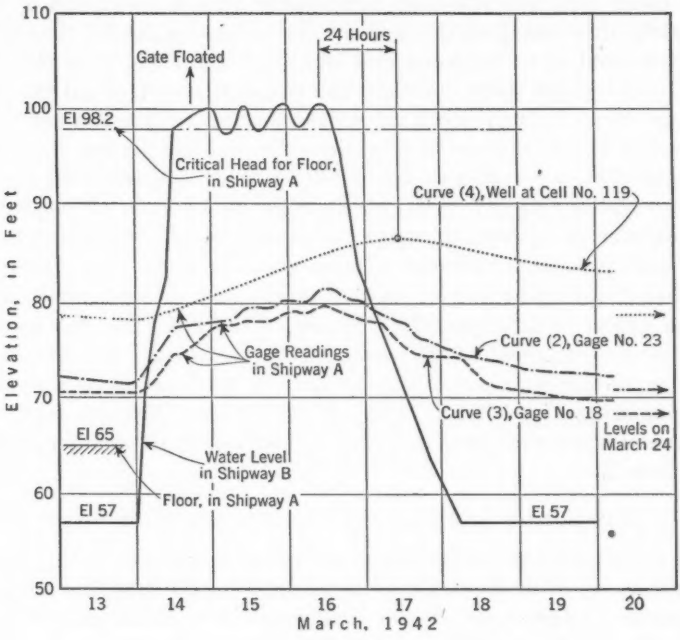


FIG. 10

water level in shipway B and those of curves (2) and (3) the piezometric levels computed from the gage readings. Curve (4) indicates the variations of the water level in the observation well. It plainly discloses the lag due to the resistance against the flow of the water into and out of the well. On account of this lag the well readings cannot be relied upon unless the hydrostatic pressure conditions change very slowly.

Fig. 9(*b*) shows the curves of equal piezometric level at the base of the marl beneath shipway B while shipway A was flooded in April, 1943. In the vicinity of the east end of the shipway the piezometric elevation ranged between 80 and 85. According to Fig. 8 the corresponding factor of safety with respect to heave ranges between 1.4 and 1.1, which is rather close to unity. In order to increase the margin of safety, seven filter wells were installed in January, 1944, in the western end of shipway B (see Fig. 9(*c*)). Their lower ends are at about El. 0. To construct these wells, 3-in. extra-heavy casings were jacked into the ground through weep holes. The lower end of each casing was capped with a steel disk. The jack pressure required to force the capped casing to a depth of about 50 ft into the stiff soil was about 12,000 lb.

After a casing was jacked to the required depth, its interior was filled with pea gravel at a compressed air pressure of about 20 lb per sq in.; then the casing was pulled. The compressed air prevented the walls of the hole from collapsing while the pea gravel was dropping out of the casing into the hole. The steel plate that plugged the lower end of the casing during jacking remained in the ground. After each well had been installed, the discharge was measured with a water meter that was attached temporarily to the upper end of the well. Immediately after installing the well the discharge was greater than 20 gal per hr. It decreased at a decreasing rate and after about one week the discharge of every well became fairly constant and equal to about 15 gal per hr. The average discharge from the weep holes was less than 1 gal per hr per weep hole. The curves of equal piezometric elevation after the installation of all the wells, in January, 1944, are shown in Fig. 9(*c*). Although all the wells are close to the eastern end of shipway B, they reduce the piezometric elevation over the entire area covered by both shipways by about 5 ft. This was due to the fact that the wells intercepted the flow of seepage at the place where it entered the soil beneath the shipway floors. In the eastern part of the shipway, where the margin of safety with respect to a heave of the floor was too narrow, the specified safety requirements are now more than satisfied. Fig. 9(*d*) shows the curves of equal piezometric head in June, 1944, while shipway A was flooded.

By comparing Fig. 10(*d*) to 10(*b*) it can be seen that the sand wells increased the minimum factor of safety of the floor of shipway B from about 1.1 to more than 2.

Deflection And Elastic Properties of The Walls

Fig. 11 represents the deflection of the crest of each of the three east-west walls immediately after the wall was acted upon for the first time by one-sided water pressure. The cause of the characteristic W-shape of the deflection curves has been discussed elsewhere.[4] Referring to Fig. 11(*a*), on November 20, 1941, the top of the fill was at about El. 100 and the gravel pocket was still filled with water. The excessive deflection of the eastern part of the south wall is due to the permanent deflection produced by the lateral pressure exerted by the clay backfill, prior to the removal of this fill and to the consolidation of the marl fill in the cells. The deflections that occurred after these events are represented by the ordinates of curve B, Fig. 11(*a*). They are greatest near the western end where the closure cells are located. The excessive deflection of the cells near the west end of the middle wall is due to the fact that the fill in these cells rests on soft mud, whereas that in the other cells rests on stiff marl. The effect of the compressibility of the base of the fill on the deflection of cofferdams has been analyzed elsewhere.[4]

The most instructive data were obtained from observations on the middle wall which is periodically acted upon by water pressure either from the north or from the south. In Fig. 12(*a*) the plain curves indicate the relation between the unbalanced horizontal pressure and the deflection for cell No. 75, at which the deflection of the middle wall was greatest. Shipway A was pumped out in August, 1941, while shipway B was still flooded (curve ab). During the first twelve days after shipway A had been completely emptied, the deflection of

cell No. 75 increased by more than 3 in., whereupon it was decided to reduce the free height of the wall in the proximity of cell No. 75 by means of a concrete sill, C in Fig. 12(*d*), with a height of 2 ft, bearing against the foot of the wall. Since the deflections continued to increase, although at a lesser rate, as shown

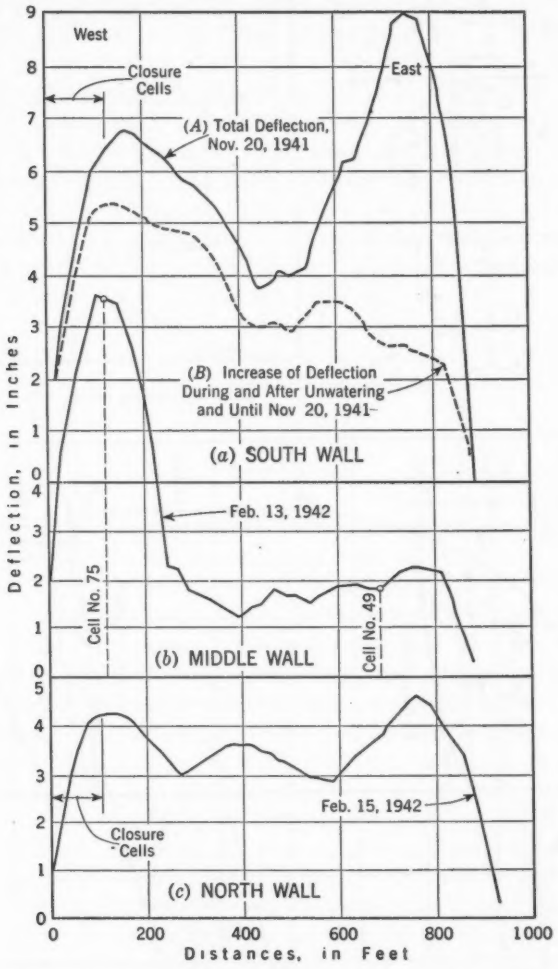


FIG. 11.—DEFLECTIONS ALONG THE CRESTS OF SHIPWAY WALLS

in Fig. 12(*b*), the movement was stopped by struts which transferred the unbalanced pressure on to the floor of shipway A.

The unwatering of shipway B, in January, 1942, caused the wall to move into the position marked d in Fig. 12(*a*). The line abb'c'cd represents the first cycle of loading produced by the application and subsequent removal of one-sided water pressure. This first cycle (August, 1941, to January, 1942) was followed by two others in March, one in August, and one in November, 1942. The crest movements produced by the first cycle in March and by that in

Deflection, y, toward the South, in inches
Unbalanced Water Pressure, in Kips per Foot
On South Side
On North Side
(a)
1 Day
7 Days
150 Days
12 Days
34 Days
Cell No. 49
Cell No. 75
(Closure Cell)
C_2 (March, 1942)
C_4 (Nov., 1942)
C_5 (April 1943)
Struts Installed
2' Sill Concreted
Time, in Days
(b)
(c)
(d)
South
North
Middle Wall
Open Fissure While A is Flooded
El 108
El 101.5
El 100
El 65
El 61
$B = 36'$
$H = 39'$
$H_1 = 40.5'$
$B_1 = 40'$
$B = 0.90\,B_1 = 36'$
$L = 25'$
$r = 25'$
$+Q$ $(+F)$
$-Q$ $(-F)$
$M(M_1)$

FIG. 12

November are represented by the hysteresis loops C_2 (blank) and C_4 (shaded area), respectively. Finally, in April, 1943, shipway A was filled while B was empty (shaded area C_5).

Cell No. 75 is one of those cells of the middle wall in which the fill rests on mud. For comparison, the effect of cyclic loading on a cell with a stiff marl base (cell No. 49) is shown in Fig. 12(*a*) by dashed lines. The deflection produced by the first application of the one-sided water pressure on either cell No. 75 or cell No. 49 was much greater than that produced by any of the following applications. Similarly, the recovery of both cells due to the first removal of the horizontal pressure was much greater than that due to the following ones. These differences indicate a considerable increase of the stiffness of the middle wall between January and March, 1942. It is most likely due to a combination of several causes, including additional consolidation of the fill in the cells caused by pumping out both adjoining shipways, increase of lock friction due to rusting, and increase of stiffness of the marl fill by aging.

The data represented in Fig. 12 permit a rough evaluation of the average modulus of elasticity of the middle wall. If τ_s represents the average shearing stress at the base of the wall and θ denotes the angular deflection of a cell, the average modulus of elasticity in shear for the cell is

$$G = \frac{\tau_s}{\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

According to the laws of mechanics the modulus of elasticity E for materials with a Poisson ratio of 0.25 to 0.5 ranges between 2 G and 2.5 G. The angular deflection of cell No. 49 due to the overturning moment M = 630 kip-ft per ft was 0.00195. On the basis of this value, the modulus of elasticity of the middle wall at cell No. 49 is approximately 190 tons per sq ft. The E-value for a clay with the stiffness of the marl ranges between 50 and 100 tons per sq ft. The E-value for a double-wall sheet-pile cofferdam is strictly equal to that of the fill. Hence, the average deflection of such a cofferdam would be from two to four times greater than that of a cellular cofferdam with equal dimensions. The difference is due almost exclusively to the lock friction in the cross-walls.

To supplement the results of the field observations the factor of safety G_s was computed for the middle wall, with respect to failure due to one-sided water pressure with one of the adjoining shipways flooded and the other empty. In a previous paper[4] it has been shown that a cellular cofferdam fails as soon as the shearing force on the neutral vertical plane of the dam becomes equal to the sum of the shearing resistance of the fill and of the friction in the locks which are located in this plane. The resisting moment M_s due to the lock friction can be computed on the assumption that the shearing resistance of the fill is zero. To compute the resisting moment M_f due to the shearing resistance of the clay it must be assumed that the lock friction is zero. The total resisting moment M_r per unit of length of the dam is then equal to

$$M_r = M_s + M_f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

It has been shown further that the overturning moment tends to pull the sheet piles on the water side out of the ground and to push those on the land

side into the ground. At the instant of failure of the cofferdam by shear the force that acts on the sheet piles becomes equal to V_r per unit of width of the sheet piles.

The coefficient of lock friction f has not yet been reliably determined. Hence, when computing the numerical value of the factor of safety G_s, two extreme values, 0.2 and 0.3, were assigned to f. Finally, it was assumed that the fill in the cells rests on a solid base. For the eastern and the middle part of the wall this assumption is justified. However, in the western part of the wall the fill in the cells rests on a highly compressible stratum of soft clay. According to theory[4] this condition reduces the factor of safety considerably and it accounts for the excessive deflection of this part of the wall, shown in Figs. 11 and 12.

If M indicates the overturning moment produced by the one-sided water pressure and M_r the resisting moment given by Eq. 4, the factor of safety of the wall is determined by the equation

$$G_s = \frac{M_r}{M} = \frac{M_s + M_f}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Computations furnished the following results:

(*a*) For the case of water pressure from the north: $M_s = 418$ ($f = 0.2$) to 625 ($f = 0.3$) kip-ft per ft; $M_f = 486$ kip-ft per ft; $M = 454$ kip-ft per ft; and safety factor $G_s = 2.00$ ($f = 0.2$) to 2.45 ($f = 0.3$). On the water side of the neutral plane the sheet piles were pulled at the instant of failure with a force of $V_r = 18.4$ ($f = 0.2$) to 22.6 ($f = 0.3$) kips per ft. On the shipway side they were pressed down with the same force. The resistance of the piles against pulling or pushing was considerably greater than these values.

(*b*) For the case of water pressure from the south: $M_s = 516$ ($f = 0.2$) to 775 ($f = 0.3$) kip-ft per ft; $M_f = 532$ kip-ft per ft; $M = 562$ kip-ft per ft; $G_s = 1.86$ ($f = 0.2$) to 2.32 ($f = 0.3$); and $V_r = 21.2$ ($f = 0.2$) to 26.5 ($f = 0.3$) kips per ft.

At the instant of failure the shearing stresses at the boundaries of the fill would still be considerably smaller than the shearing strength of the clay. Hence, the failure would occur while the distribution of the pressure on the base of the fill is still fairly uniform. In both the loading cases (*a*) and (*b*) the overturning moment M produced by the one-sided water pressure is either slightly greater (if $f = 0.2$) or considerably smaller (if $f = 0.3$) than that part, M_s, of the total resisting moment of the wall which could be carried, if necessary, without the assistance of the shearing strength of the fill in the cells.

Observations Concerning the Sheet-Pile Walls of the Cells

The steel shell of the cells consists of straight-web, steel sheet piles with a web 1/2 in. thick. Fig. 13 is a section through a lock and Fig. 14 represents the relation between lock tension and unit elongation in the direction of the tensile force. The height of the specimen measured in the direction of the locks was 2 in. One of the ten specimens tested failed in the web, and the others failed

in the finger or thumb at stresses ranging between 17,750 and 20,950 lb per in. According to Fig. 14 the failure strain is about 3.5%. Because of the small height of the specimen, the initial play in the lock was negligible. The initial play in the locks of a row of sheet piles is about 1/4 in. which increases the strain at failure in the direction of the hoop tension from 3.5% to about 5%. It should be noticed that the stress-strain curve in Fig. 14 is almost horizontal between a stress of about 5,000 and 10,000 lb per in.

FIG. 13

The process of driving the sheet piles was described by Mr. Jansen.[6] A short time before shipway A was pumped out, it was discovered that a pair of sheet piles was driven out of groove, below El. 60, in each of four cells of the middle wall. Considering the fact that the cofferdams comprised 125 cells and that the marl was stiff, this record is very good. The damage was repaired easily by concreting the shipway floor to a distance of 20 ft from each end of each defective section in an east-west direction and to a distance of 20 ft to the north of it in advance of the remainder of the shipway floors. This operation was preceded by driving all the piles to be located within these areas.

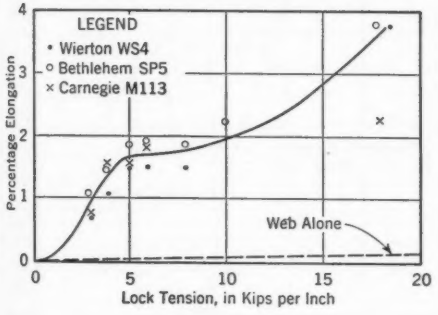


FIG. 14

To form some conception of the deformation of the steel walls of the cells, the following observations were made by the Dravo Corporation: Measurement of the length of the upper edge of the cross-walls of every cell after each operation which might change the tension in these walls; survey of the shape of two Y-piles and of the sheet piles at the middle third points of the arcs on the inside of cells No. 15 and No. 31 in the south wall, immediately after shipway A was pumped out and a few weeks later, after the piles for supporting the crane track south of the wall were driven; survey of the shape of two Y-piles, of the center sheet of the arc and of horizontal sections through the arc at El. 68 and El. 111 on cells No. 5 (south wall), Nos. 72 and 73 (middle wall), about three weeks after shipway A was pumped out and on cell No. 120 (north wall) about two weeks after unwatering shipway B; survey of the entire shape of cell No. 53 (middle wall), two weeks after shipway B was pumped out; survey of the south face of cells Nos. 113, 116, and 124 of the north wall and of both faces of the cell No. 75 (middle wall), four months after shipway B was pumped out.

Table 2 contains the average elongation of the upper edge of the cross-walls, as well as the range of the extreme values. The table shows that the driving of wood and sand piles nearly doubled the elongation and increased the elongation of most of the cross-walls to more than 2%. It should be noticed that the driving of sand piles increased the elongation much less than the driving of

TABLE 2.—ELONGATION OF THE UPPER EDGE OF THE CROSS-WALLS

Cause of elongation	CELLS NOS. 1 TO 34		CELLS NOS. 41 TO 80		CELLS NOS. 86 TO 125	
	Average	Range	Average	Range	Average	Range
Filling	3.90	6.0 to 1.3	4.24	7.5 to 0.75	4.32	6.0 to 1.1
Driving wooden piles	0.67	4.1 to 1.5	2.41	7.5 to −1.4	3.38	6.0 to −0.75
Time effect	1.24	2.1 to 0.4	2.36	7.5 to 0	0.63	1.5 to −0.6
Driving sand piles	0.60[a]	....	0.60	1.6 to 0	0.63	1.5 to −0.6
Total:						
Inches	6.41	....	9.61	....	8.96	....
Percentages	1.9	....	2.42	....	2.26	....

[a] Estimated

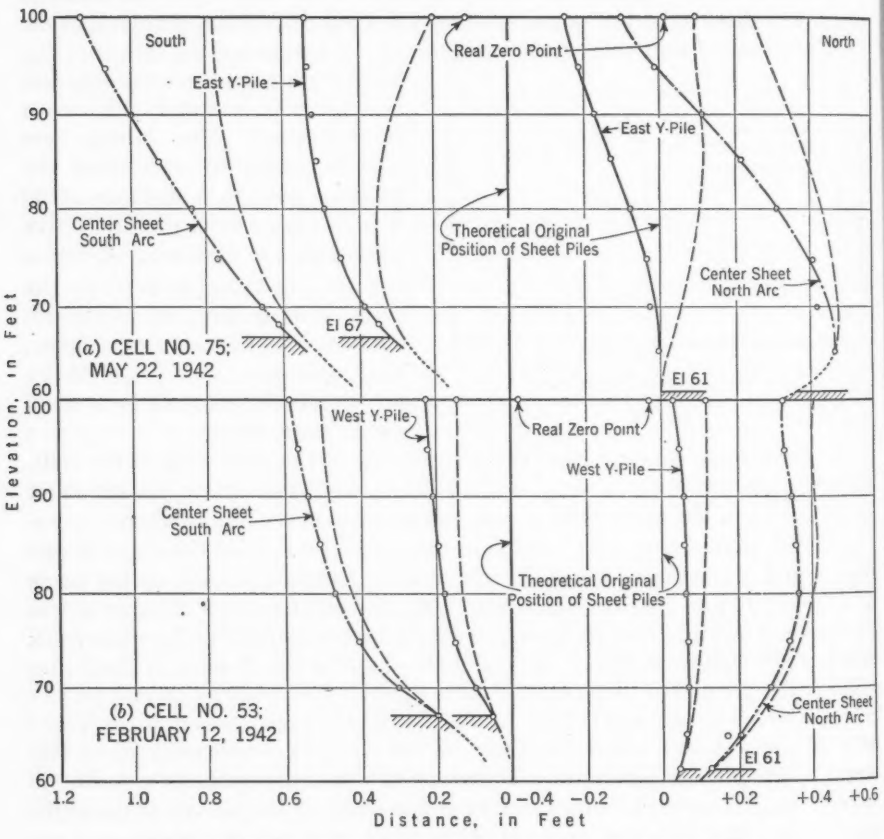


FIG. 15.—CHANGE IN THE SHAPE OF THE CELLS OF THE MIDDLE WALL CAUSED BY AN UNBALANCED HORIZONTAL PRESSURE

wood piles; yet the volume displaced by the sand piles is greater than that displaced by the wood piles. These facts suggest that the driving of wood piles increased the lock tension to a value greater than 5,000 lb per in. at which the stress-elongation curve in Fig. 14 flattens out. They also suggest that the lock

tension prior to driving the wood piles was appreciably smaller than 5,000 lb per in.

The change of the shape of the cells caused by an unbalanced horizontal pressure is illustrated by Fig. 15. In each case the solid line indicates the shape and position of the outer edges of one of the cross-walls and the dash-dot line indicates the shape and position of the center sheet of the arcs, about four months after the unbalanced water pressure on the middle wall was removed by unwatering shipway B. Since the elastic recovery (curves cd and c_1d_1 in Fig. 12 (*a*)) was slight compared with the preceding deflection, the shape of the cells is almost identical to that of the cells prior to the removal of the unbalanced pressure. The position of the sheet piles was measured by offsets from the center line of the adjoining shipways. The surveyed points are indicated by small circles. It should be noted how close the real original position of the edges of the cross-walls (real zero point) is to that required by the construction drawings (origin of the abscissa). The thin dashed lines indicate the estimated position of the sheet piles prior to the application of the unbalanced pressure.

Over one half of the height of each cell between El. 80 and El. 100, the center sheet piles of the two arcs are almost parallel to each other. Hence, from the top of the cells down to about midheight, the lock tension was about the same everywhere; and below midheight, it decreased to zero at the base of the cells. This condition indicates that the hoop tension also decreased below midheight.

Fig. 16 is a horizontal section through the south arc of cell No. 124 at El. 65 surveyed after shipway B had been pumped out. Originally the arc had a radius r of about 24.4 ft. At the time of the survey the radius of curvature decreased from about 0.92 r at the crown to about 0.68 r on both sides. Since the tension at a given elevation is the same in every lock, the decrease of the radius of curvature from the crown toward the hinges indicates an increase of the lateral unit soil pressure in the same directions. This condition was typical for most of the cells, but in some of them the radius was smallest in the proximity of the crown. Hence, the distribution of the soil pressure over the arcs scatters very considerably from the statistical average. The same conclusion applies to the shape of the cross-walls. Since the average strain in the sheet pile depends to a considerable extent on the initial play in the locks, which is unknown, reliable information concerning the lock tension can be obtained only by means of strain-gage measurements on the web. The provisions for such measurements must be made before the sheet piles are driven. On account of imperfect fit in the locks, only the averages of readings on a great number of pairs of reference points can be relied upon. A practicable method for measuring the lateral pressure in the fill of the cells has been suggested elsewhere.[10]

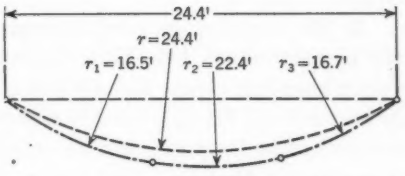


FIG. 16.—SOUTH ARC, CELL NO. 124; EL. 65; MAY 23, 1942

[10] *Proceedings*, Am. Soc. C. E., June, 1945, p. 980.

Summary

1. Comparison between the cost of the shipways and the cost of other recently constructed dry docks showed that the design described in this paper was remarkably economical. Savings were achieved by providing the shipways with walls of the cellular type and by taking advantage of local soil conditions.

2. On account of the radical departure from conventional methods the design was inevitably based on various more or less uncertain assumptions. However, by systematic field observations during construction it was possible to correct the initial errors and to modify the design during construction in accordance with the findings. This procedure avoided the risk of a failure in spite of the novelty of the design, and the cost of the modifications did not exceed a small fraction of the total savings.

3. The observations during construction showed that the deflection of the walls was greater than the designers had anticipated. This condition was remedied by two independent measures. The stiffness of the fill in the cells was increased effectively by compaction and drainage, accomplished by means of sand piles. The unbalanced horizontal pressure on the outside walls was reduced by replacing the clay backfill by a drained gravel backfill.

4. The deflection observations confirmed the theoretical conclusion that the deflection of long cofferdams with fixed ends, resting on a firm base, should be greatest at a short distance from each end. They also showed, in accordance with theory, that the presence of a highly compressible layer beneath the fill in the cells increases the deflection very considerably, everything else being equal.

5. The relation between the unbalanced horizontal pressure on a wall and the corresponding deflection was found to be very similar to the relation between shearing stress and shearing strain for cohesive soils. The deflection increases more rapidly than the horizontal pressure, and at constant pressure the deflection increases at a decreasing rate. However, by trial computation it was found that the sheet-pile enclosure contributes very materially to the stiffness of the entire structure.

6. The factor of safety of the middle wall with respect to a failure by shear on a vertical plane was estimated by assuming that the coefficient of lock friction had a value between 0.2 and 0.3. On this assumption it was found that the factor of safety of that part of the wall which rests on solid marl ranged between 1.8 and 2.4. It was also found that the overturning moment had almost no effect on the distribution of the pressure on the base of the wall. This distribution was practically uniform. For the western part of the wall the factor of safety was considerably smaller than the foregoing values because this part of the wall rests on a very compressible base.

7. The results of a survey of the shape of the cells, after filling them with marl, showed that the hoop tension in the sheet-pile enclosures has almost the same value at any elevation within the upper two thirds of the height of the cells. Within the lower third it decreases in a downward direction to almost zero at the level of the base. Therefore, the unit elongation of the upper edge

of the cross-walls is practically equal to the maximum horizontal unit elongation of the sheet-pile enclosure of the cells.

8. While bearing piles and sand piles are being driven into the fill in the cells the lock tension is increased by an amount that cannot be predicted reliably. To make sure that the lock tension remains within safe limits the stretch in the upper edge of the cross-walls was measured repeatedly while driving the piles. The results of the measurements combined with those of previous tension tests with short sheet-pile sections indicated that the lock tension never exceeded 10 kips per in.

9. The design of the shipway floors was based on the assumption that the flow of the ground water toward the weep holes reduced the hydrostatic uplift on the base of the marl stratum which contained the piles; but the amount of reduction was not known. To observe whether the floors were safe with respect to heave, gages were installed which registered the water pressure at about the level of the points of the piles.

10. While the shipways were pumped out for the first time, gage readings were made at regular intervals. The readings showed that the safety of the floors with respect to heave will be adequate at any operating stage of the shipway, except at the eastern end of the floor of shipway B. By installing seven 3-in. filter wells beneath that part of the floor the piezometric head was reduced over the entire area covered by the shipways by about 5 ft. This local remedy served its purpose and it was much more economical than the customary method of preventing a heave by increasing the thickness of the floors.

Acknowledgments

The submerged shipways described in this paper were a project of the Bureau of Ships, U. S. Navy, under the general supervision of O. L. Cox, Admiral, U. S. Navy, and appreciation is expressed to the Navy Department for permission to publish the data contained in the paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRESSES IN THE LININGS OF SHIELD-DRIVEN TUNNELS

Discussion

BY G. N. ALEXANDER

G. N. ALEXANDER,[36] ESQ.[36a]—As there is no generally accepted method for designing tunnel linings, the author's analysis is very welcome in providing a rational method for determining what are generally termed passive pressures. Although there may be good grounds for maintaining that successful precedent, where such exists, is the best guide to tunnel lining design, progress depends in part on the promulgation of such theories. However, until knowledge of active pressures advances, elaborate theories regarding passive pressures tend to be academic.

The author rightly criticizes the "classical" theory[2] given by S. Johannesson and the late B. H. M. Hewett, Members, Am. Soc. C. E., because it does not consider that the deflection of the lining is sufficient to give rise to stresses. This theory depended on an assumption that, provided the total passive pressure determined from Rankine's theory is not exceeded, bending stresses due to earth pressure need not be considered. This may lead to unsafe rather than conservative designs. The late Walter C. Parmley,[37] M. Am. Soc. C. E., directed attention to the weakness of their contentions and made an analysis based on an assumed elliptical deflection curve with passive pressures depending on the deflections in the lateral quadrants. Such methods require what is variously termed the "soil constant" (M. A. Drucker), "soil compression conconstant" (Anders Bull, M. Am. Soc. C. E.), and "coefficient of horizontal soil reaction" (Karl Terzaghi, M. Am. Soc. C. E.). Unfortunately, experimental evidence concerning the value of the "soil constant" K is extremely meager and any designer using it must do some "guestimating." It therefore seems desir-

NOTE.—This paper by Anders Bull was published in November, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1945, by J. A. Van den Broek; February, 1945, by Nathan D. Brodkin, M. A. Drucker, and Sigvald Johannesson; May, 1945, by Jacob Feld; June, 1945, by Leon Beskin, and D. P. Krynine; and September, 1945, by A. A. Eremin.

[36] Designing Engr., Melbourne and Metropolitan Board of Works, Melbourne, Victoria, Australia.

[36a] Received by the Secretary September 10, 1945.

[2] "Shield and Compressed Air Tunneling," by B. H. M. Hewett and S. Johannesson, 1st Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1922, p. 74.

[37] Discussion by Walter C. Parmley of "Notes on Tunnel Lining for Soft Ground," by S. Johannesson and B. H. M. Hewett, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 1880.

able to derive an expression to show rapidly the effect of varying the factors that influence passive pressures. Some consideration must also be given to the assumptions made.

The author criticizes Mr. Drucker[3] (see Section 3) and other exponents of theories (such as M. G. Spangler,[10] M. Am. Soc. C. E., in Section 28) who use a uniform vertical load to determine passive pressures. For analytical investigations loading may be expressed by using rectangular or polar coordinates. It would appear that the loading assumptions made may depend on the system chosen—a scarcely defensible procedure. If rectangular coordinates are used it is generally assumed that the horizontal pressure is equal to k times vertical pressure, which may be termed case I. If such a loading is expressed in polar coordinates tangential tractions are involved, but the author, by deductive processes, disposes of these which may be termed case II. Fig. 27 illustrates

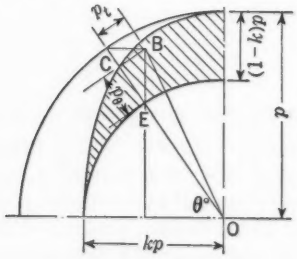


FIG. 27.—ACTIVE PRESSURE

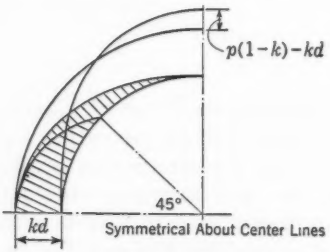


FIG. 28.—PASSIVE PRESSURE

the two systems and the relative magnitude of the tangential tractions, at θ. Of the radial pressure OC, only the variable part EC is effective in producing distortion and hence effective passive pressures. As $\overline{CB} = \overline{CE} \cot \theta$ it cannot be neglected on the score of the relative magnitude.

To determine the relative importance of radial and tangential loadings it may be shown that, for a constant vertical pressure of p_v, the value of M_θ is given for case I by

$$M_\theta = \frac{p_v r^2 (1 - k) \cos 2\theta}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

For case II, for a radial pressure only, given by

$$p_\theta = p (1 - k) \sin^2 \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

the corresponding moment can be determined most readily by considering a pair of equal but opposite diametrical loads. Thus, if $P_\theta = p_\theta r \, d\theta$, M'_ϕ due to P_θ, with $\theta > \phi$, is given by[38]

$$M'_\phi = \frac{P_\theta r}{2} \left(\sin \overline{\theta - \phi} - \frac{2}{\pi} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (60)$$

[3] "Determination of Lateral Passive Soil Pressure and Its Effect on Tunnel Stresses," by M. A. Drucker, *Journal of the Franklin Institute*, May, 1943, p. 499.

[10] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 153*, Eng. Experiment Station, Iowa State College, Ames, 1941, p. 61.

[38] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., 1940–1941, Pt. II, p. 82.

Hence the bending moment at ϕ due to distributed load is given by

$$M_\phi = \frac{p\,(1-k)\,r^2}{2}\int_{\theta=\phi}^{\theta=\pi+\phi} \sin^2\theta\left(\sin\overline{\theta-\phi}\,\frac{2}{\pi}\right)d\theta = \frac{p\,(1-k)\,r^2\cos 2\phi}{6} \quad (61)$$

Thus, the tangential tractions produce half as much moment as the radial loads and clearly should not be neglected unless more evidence can be produced to show that such tangential loadings as implied by the use of the relation $p_h = k\,p_v$ do not exist. To determine the deflections the differential curve of the ring may be used,[39] thus:

$$\frac{d^2d}{ds^2} + \frac{d}{r^2} = -\frac{M}{E\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (62)$$

Substituting the value for M given by Eq. 61 and solving, the deflection at any point is

$$d_\phi = \frac{p\,r^4\,(1-k)\cos 2\phi}{18\,E\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (63)$$

That is, for case I the value is increased by 50%.

Passive Pressures.—Deflections and moments calculated on active pressures alone will not generally be realized due to the effect of passive pressures. Various assumptions are made as to how, where, and when these passive pressures act. The use of a soil constant K seems necessary but its use involves crude approximations, and consolidation effects may mask any elastic effect. Grouting also may have considerable significance. The author assumes that passive pressures are directly proportional to the radial deflections outside the settled position of the undeflected ring. The writer also assumes that the active pressure is unchanged at the crown but the movement of the tunnel in a vertical direction may be disregarded in determining passive pressures due to deflection of the lining. In this case the active pressure $p\,k$ (Fig. 27) of the horizontal diameter is increased by a passive pressure, $K\,d$ (Fig. 28), in which d is the net maximum radial deflection. For the analytical determination of moments this passive pressure $K\,d$ can be subtracted over the entire circumference from the net active pressure causing distortion used previously, leaving a pressure difference between the crown and sides of $p\,(1-k) - K\,d$ (Fig. 28). At first it may appear safest to consider that passive pressures are limited to the two lateral quadrants. However, such pressures can be maintained only by the deflection of the lining due to the excess of vertical over horizontal active pressures. This requires continuous contact between earth and lining and it is presumed that the final pressure curve is continuous. Therefore, the writer assumes a final pressure distribution curve of the same form as that for active pressure. The passive pressures additional to those which may undoubtedly be assumed to occur over the lateral quadrants will cause no significant distortion to the lining, as each of the four quadrants is equally loaded with these additional pressures (Fig. 28).

[39] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., 1940–1941, Pt. II, p. 103.

Hence, the net deflection d due to active and passive pressures, assuming the net loading is due to radial loads only, is as follows:

$$d = [p\,(1 - k) - K\,d]\,\frac{r^4}{18\,E\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots (64a)$$

Therefore,

$$d = \frac{p\,(1 - k)\,r^4}{18\,E\,I}\,\frac{1}{1 + \dfrac{K\,r^4}{18\,E\,I}} = d_a \times \frac{1}{1 + n} \quad \ldots\ldots\ldots\ldots (64b)$$

In Eq. 64*b*, $n = \dfrac{K\,r^4}{18\,E\,I}$ and the maximum active deflection is $d_a = \dfrac{p\,(1 - k)\,r^4}{18\,E\,I}$; or

$$d = \frac{p\,(1 - k)\,n}{K\,(1 + n)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (65)$$

Thus the dimensionless term $\dfrac{K\,r^4}{18\,E\,I}$ may be evaluated to give the effect of the passive pressures. The factors in this term deserve consideration. Clearly the most significant is r and large diameter tunnels may make much more use of passive pressures unless tie rods and roadways reduce the deflection. The moment of inertia I may have only a fraction of its theoretical value due to flexibility at the joints, especially in a cast-iron tunnel where longitudinal joints are not staggered. This flexibility (if strength is not impaired) therefore may be an advantage. It would be advisable to determine the value of $E\,I$ for a ring by experiment if Eq. 65 were to be used.

There appear to be good grounds for the author's contention concerning the advantages of steel, especially as the governing stresses in the conventional cast-iron lining are tensile. The provision for jacking stresses and the apparent unwillingness of designers to depart from customary methods would seem to be the major reason for the continued use of cast iron.

Therefore, it appears that, although it is desirable that more rational methods be applied to tunnel design than heretofore, much more experimental evidence is necessary to enable the variables to be determined before such an elaborate theory as that proposed by the author is necessary. The writer suggests that for all cases, except large shallow tunnels, the ratio of passive to active pressures, deflections, and moments is given approximately by the ratio, $\dfrac{K\,r^4}{K\,r^4 + 18\,E\,I}$.

The author is to be congratulated on his extensive mathematical work and it is to be hoped that his paper will accelerate the accumulation and dissemination of data on soil pressure on tunnels to enable a theory to be developed which can be substantiated by experimental data.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

COLUMN FORMULAS

Discussion

BY LOUIS BALOG

LOUIS BALOG,[19] ESQ.[19a]—Imperfections in technical processes, and economic considerations, limit the precision attained in making and shaping of metals and in fabricating and erecting structural members. Homogeneity and physical properties of materials, the geometric accuracy of members and structures, the conformity to the contemplated points of application, and the character of the calculated forces depend on practices that must be justified economically for various types of structures. Since all these circumstances influence the carrying capacity of a compression member, the derivation of empirical column formulas, of universal validity, is impossible.

For absolutely ideal conditions the $\left(\sigma_{cr} - \frac{l}{i}\right)$-curve is composed of Euler's hyperbola and of a horizontal line $\sigma_{cr} = \sigma_y$, as shown in Fig. 5. The most exacting laboratory tests have failed to conform to this curve, due to the sensitivity of columns to deviations from ideal conditions. Study of the effects of tolerances in metal and of rolled shape characteristics, and experience with the accuracy of statical calculations and with that of shop and erection work, lead to the reduction of the theoretically ideal curve to curves shown below it in Fig. 5, which curves indicate the practical carrying capacity of carbon-steel and silicon-steel columns in bridge and structural work. The deviations between the ideal and actual conditions are so great in this case that the mathematical criteria, set up by the author for establishing the shape of column curves, seem to be of little practical consequence.

The practical validity of the formulas in the paper depends not so much on the primary requisites defined by Mr. Osgood as on the selection of the arbitrary constants they contain. These constants can indeed be so selected that the formulas will approximate special conditions more or less satisfactorily; but the restrictions on the constants, to obtain curves representing monotoni-

NOTE.—This paper by William R. Osgood was published in December, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1945, by L. E. Grinter, George Winter, and Jonathan Jones; March, 1945, by Edward Godfrey; May, 1945, by Ralph E. Spaulding, and James H. Cissel; and June, 1945, by Leon Beskin.

[19] Cons. Engr., New York, N. Y.

[19a] Received by the Secretary July 3, 1945.

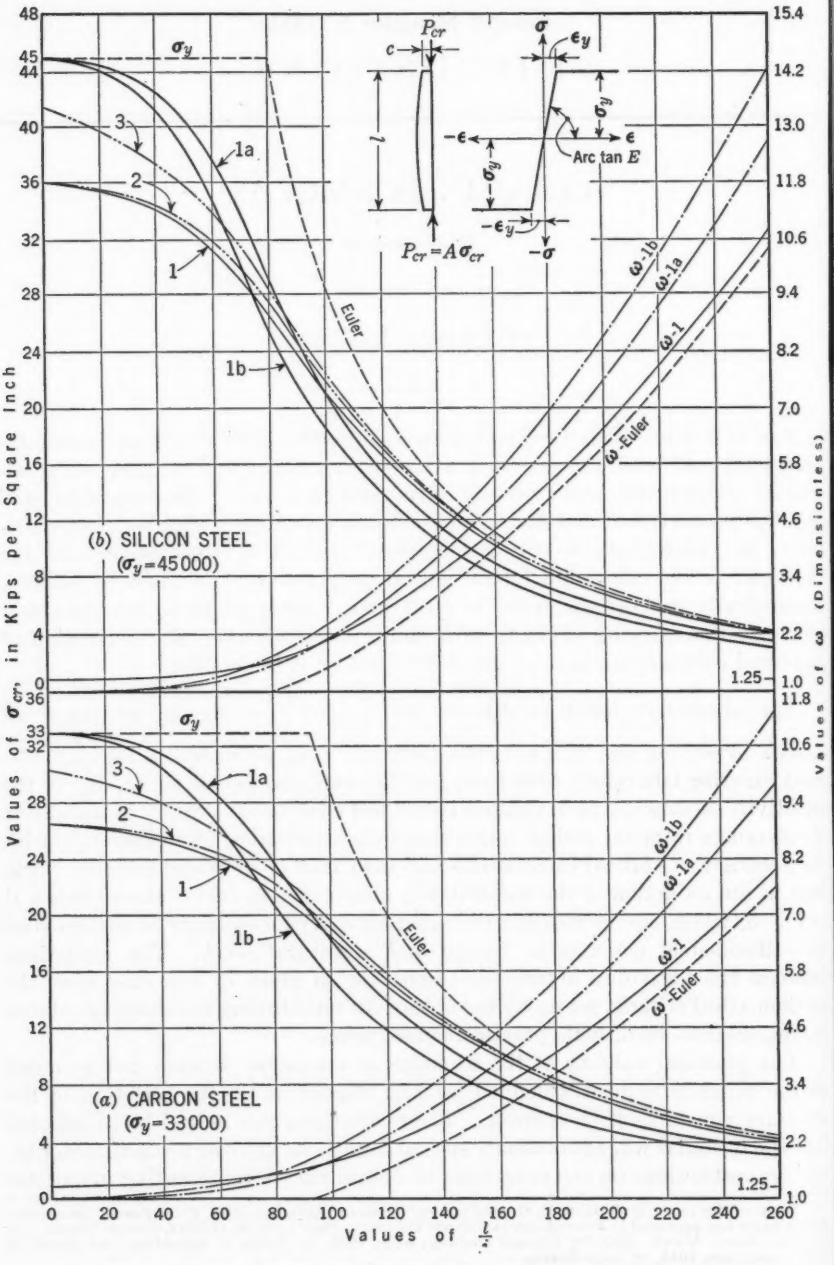


FIG. 5.—COLUMN CURVES FOR CARBON-STEEL AND SILICON-STEEL STRUCTURES
(E = 29,400,000 Lb per Sq In.)

cally decreasing functions (the first derivatives of which are continuous) are irrelevant requirements from point of view of practical design.

Although columns have been designed by the use of empirical formulas, the general requirements of engineering practice could never be met with such formulas. This alone would be a sufficient reason for their rejection; but empirical formulas also hinder progress in design. The problem, therefore, is not to fit a formula adequately to a series of column tests (which seldom represent actual structural conditions), but to fit a theory to reality. Rational theories, by which the effects of varying all characteristics that enter into the problem can be measured, should be the basis of design.

In the "Introduction" the author states that

> "It would be possible to design by means of the double-modulus theory * * * but * * * most engineers would undoubtedly prefer to use an empirical formula based on tests."

Bridge and structural design practice does not support this opinion. Empirical formulas, based on tests, have been abandoned and theoretical formulas, in the derivation of which the modulus variation is neglected, are in general use. No other conclusion can be drawn from the widely scattering results of column tests than that only theoretical formulas are rational. Tests have failed to furnish data for the basic constants of the theoretical formulas—the eccentric ratio a and the reduced length l_o (Eq. 2)—on which two factors the validity of any practical column formula depends. Values for both of them can be determined, however, from the characteristics of actual structures and from theoretical considerations. It is quite certain that imperfections in testing procedure render such experiments useless. Extremely careful tests of small specimens, to determine the limits of validity of some basic theoretical assumption, constitute the reasonable experimental investigation.

Inasmuch as axially loaded columns do not exist in an ideal sense, only eccentric loading needs to be considered. A distinction must be made between small eccentricities ($a < 0.01$) and large eccentricities ($a > 0.1$); and actual structural columns belong in the latter category.

Numerous difficulties hinder the mathematical expression of the yielding process; and therefore the actual stress-strain relation and the actual shape of the deflection line can be considered only in graphic-analytic solutions of the problem.[20,21,22,23,24,25] The labor involved in these methods restricts their practical use to the most unusual investigations. The same is true of the approximate methods in which some assumption is made for the deflection line and in which the stress-strain relation is replaced by a combination of straight

[20] "Untersuchungen über Knickfestigkeit," by Theodor von Kármán, *Mitteilungen über Forschungsarbeiten*, V.D.I., Heft 81, 1910.

[21] "Die Stabilität zentrisch und exzentrisch gedrückter Stäbe aus Bausthal," by E. Chwalla, *Sitzungsbericht der Akademie der Wissenschaften in Wien*, Abteilung IIa, July, 1928, p. 469.

[22] Rept., 2d International Cong. for Bridge and Structural Eng., Vienna, September 24–28, 1928, p. 608.

[23] Final Rept., 1st Cong., International Assn. for Bridge and Structural Eng., Paris, May 19–25, 1932, p. 53.

[24] "Über die experimentelle Untersuchung des Tragverhaltens gedrückter Stäbe aus Bausthal," by E. Chwalla, *Der Stahlbau*, Vol. 7, 1934, p. 17.

[25] "Theorie des aussermittig gedrückten Stäbes aus Bausthal," by E. Chwalla, *ibid.*, pp. 161–165, 173–176, and 180–184.

and parabolic lines.[26,27,28,29,30,31,32,33] In spite of their limited practicability of direct application in design these theoretical investigations are of much greater practical value than the tests made to determine empirical column formulas.

Diagrams containing the curves representing the functional relations,

$$\frac{l}{i} = f(\sigma_{cr}, a) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

arranged according to parameter a (see Fig. 6), facilitate the clear visualization of the behavior of compression members as influenced by the characteristics

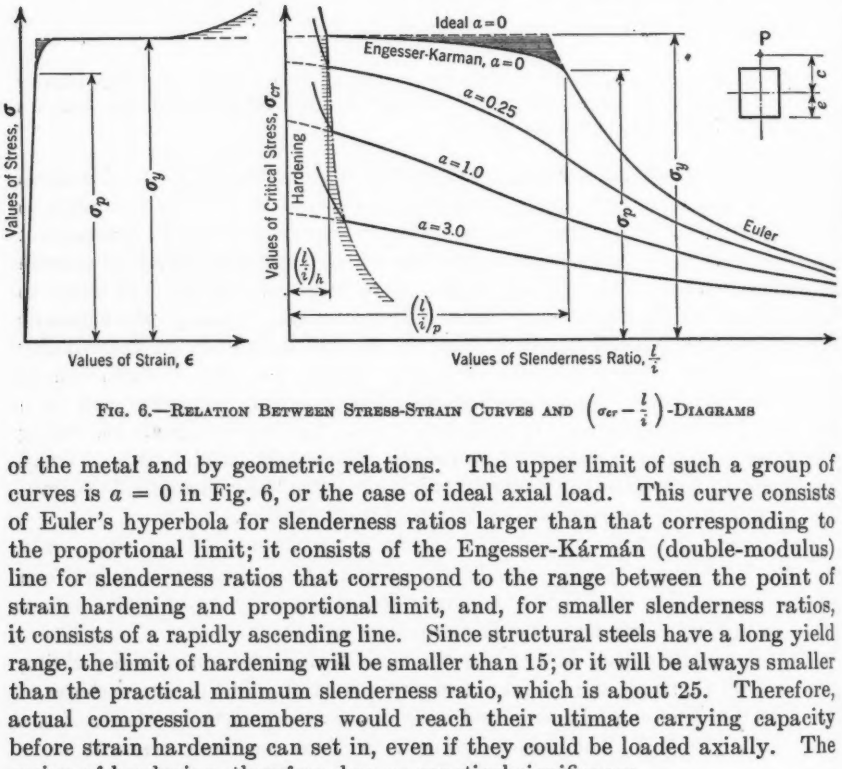


FIG. 6.—RELATION BETWEEN STRESS-STRAIN CURVES AND $\left(\sigma_{cr} - \frac{l}{i}\right)$-DIAGRAMS

of the metal and by geometric relations. The upper limit of such a group of curves is $a = 0$ in Fig. 6, or the case of ideal axial load. This curve consists of Euler's hyperbola for slenderness ratios larger than that corresponding to the proportional limit; it consists of the Engesser-Kármán (double-modulus) line for slenderness ratios that correspond to the range between the point of strain hardening and proportional limit, and, for smaller slenderness ratios, it consists of a rapidly ascending line. Since structural steels have a long yield range, the limit of hardening will be smaller than 15; or it will be always smaller than the practical minimum slenderness ratio, which is about 25. Therefore, actual compression members would reach their ultimate carrying capacity before strain hardening can set in, even if they could be loaded axially. The region of hardening, therefore, has no practical significance.

[26] "Die Knicksicherheit von an beiden Enden gelenkig gelagerten Stäben aus Konstruktionsthal," by M. Roš and J. Brunner, *Bericht No. 13* der EMPA und der Gruppe VI der TKVSB, Zürich, August, 1926.

[27] *Proceedings*, 2d International Cong. for Applied Mechanics, Zürich, September 12–17, 1926, pp. 368–378.

[28] "Die Bemessung zentrisch und exzentrisch gedrückter Stäbe auf Knickung," Rept. of the 2d International Cong. for Bridge and Structural Eng., Vienna, September 24–28, 1928, p. 282.

[29] "La stabilité des barres comprimées par des forces excentrées," by M. Roš, Preliminary Rept., 1st Cong., International Assn. for Bridge and Structural Eng., Paris, May 19–25, 1932, p. 57.

[30] "The Strength of Steel Columns," by H. M. Westergaard and William R. Osgood, *Transactions*, A.S.M.E., Vol. 49–50, Pt. I (1928), APM-50-9, pp. 65–80.

[31] "Die einseitige Druck bei Stäben aus Bausthal," by Friedrich Hartmann, Final Rept., 1st Cong., International Assn. for Bridge and Structural Eng., Paris, May 19–25, 1932, p. 40.

[32] Zeitschr. d. ÖIAV, 1933, H.11–12.

[33] Knickung Kippung Beulung, Franz Deuticke, Wien, 1937, p. 43.

The shape of the Engesser-Kármán (double-modulus) line depends on the shape of the stress-strain diagram between the proportional limit and yield stress. An infinite variety of double-modulus curves is possible between these two values for the same steel. This indicates that a reasonably assumed shape for this curve is just as close to reality as any of those which correspond to a certain actual stress-strain diagram. Comparative examinations of the effect of the variations of the double-modulus curve on the values of the carrying capacity of columns, of the effects of the practically unavoidable eccentricities of the load ($a > 0.1$), and of the fluctuation in the values of σ_p and σ_y, indicate that the double-modulus theory has no practical significance in bridge design or in structural design generally. Therefore, it is not only the simplest procedure, but also the most reasonable, to base structural column formulas on the ideal stress-strain relation.

The ideal stress-strain variation (see stress-strain diagram in Fig. 5), and the validity of the classical theory of the yielding process, are the fundamental assumptions of practical column investigations. It is assumed that yielding begins when the stress in the extreme fiber of the section reaches the yield stress of the tension-test specimen subjected to the standard tension test, and extends continuously with continuously increasing load.

Actually the stress-strain diagram is different for tension and compression, for increasing and decreasing load. It is also curved beyond the proportional limit, and the yielding process is characterized by the formation of flow layers caused by the superimposed bending stresses resulting from the unavoidable eccentricity of the loading.[34] Yielding, therefore, does not progress continuously with an increasing load but in layers, and this fact influences the stress distribution in the section and brings the shape of the section into play. The consideration of flow layers is generally neglected in present-day structural design. The curved portion of the stress-strain diagram between the proportional limit and yield stress is short for structural steels. The fluctuation of σ_y for specimens of the same steel is often larger than the length of this region. The consideration of these conditions indicates that, for structural steel columns, the foregoing idealizing simplifications are justified. Based on them, and on Bernoulli's hypothesis that originally plain sections remain plain and normal to the longitudinal column fibers after bending, the carrying capacity of compression members is adaptable to a theoretic evaluation that is entirely satisfactory for structural design.

In view of the foregoing simplifying assumptions, three different stress conditions are possible in a rectangular column section: (*a*) The condition corresponding to purely elastic deformation; (*b*) the stress pattern when there is a plastic area at the compression side; and (*c*) the stress pattern when there is a plastic area at both compression and tension sides. ("Compression side" or "tension side" denotes that side of the section in which the stress due to bending alone is compression or tension—quantities referring to these sides will be designated by subscripts *c* and *t* herein.) For the three different stress conditions of the center section (*a*), (*b*), and (*c*), the following formulas give

[34] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931, p. 104.

the carrying capacity:[35,36]

Case (*a*)—

$$\left(\frac{l}{i}\right)^2 = \frac{\pi^2 E}{\sigma_{cr}}\left(1 - \frac{a\,\sigma_{cr}}{\sigma_y - \sigma_{cr}}\right) \ldots\ldots\ldots\ldots\ldots\ldots (54a)$$

Case (*b*)—

$$\left(\frac{l}{i}\right)^2 = \frac{\pi^2 E}{\sigma_{cr}}\left[1 - \frac{a\,\sigma_{cr}}{3\,(\sigma_y - \sigma_{cr})}\right]^3 \ldots\ldots\ldots\ldots\ldots (54b)$$

Case (*c*)—

$$\left(\frac{l}{i}\right)^2 = \frac{\pi^2 E}{\sigma_{cr}}\sqrt{\frac{\sigma_{cr}}{\sigma_y}\left(\frac{\sigma_y}{\sigma_{cr}} - \frac{\sigma_{cr}}{\sigma_y} - \frac{2\,a}{3}\right)^3} \ldots\ldots\ldots\ldots (54c)$$

For the solution of Eqs. 54 only the values of E and σ_y are necessary. These formulas apply to any kind of structural steel. Eq. 54*a* is valid until the yield stress in the extreme fiber is reached; Eq. 54*b* is valid until $a \leqq 3\left(1 - \frac{\sigma_{cr}}{\sigma_y}\right)$; and Eq. 54*c* is valid until $a \geqq 3\left(1 - \frac{\sigma_{cr}}{\sigma_y}\right)$.

The practical carrying capacity, or the lower limit of the carrying capacity of columns, is defined by the load at which the yield stress is reached in the extreme fiber. Eq. 54*a* gives this lower limit on which design is usually based. It is valid for any shape of section and gives practically identical results with the "secant formula":

$$\sigma_{cr} = \frac{\sigma_y}{1 + a \sec \frac{l}{2\,i}\sqrt{\frac{\sigma_{cr}}{E}}} \ldots\ldots\ldots\ldots\ldots\ldots (55a)$$

Eq. 55*a* must be solved by the cumbersome trial-and-error method; but, transformed into the form,

$$\frac{l}{i} = 2\sqrt{\frac{E}{\sigma_{cr}}}\,\text{arc cos}\,\frac{a\,\sigma_{cr}}{\sigma_y - \sigma_{cr}} \ldots\ldots\ldots\ldots\ldots\ldots (55b)$$

it is convenient for calculations because tables for circular cosines for radian arguments are readily available; the arc of the cosine can be read from such tables, and Eq. 55*b* can be solved directly.

The upper limit of the carrying capacity is defined by the load that produces the critical equilibrium condition. Eqs. 54*b* and 54*c* give these values for rectangular section.

Rigorous investigations[37] have shown that the upper limit of the carrying capacity is appreciably influenced by certain cross-sectional shapes. A formula by which the sectional form can be considered is:[38,39]

$$\left(\frac{l}{i}\right)^2 = \frac{\pi^2 E}{\sigma_{cr}}\left(1 - \mu_1\frac{a\,\sigma_{cr}}{\sigma_y - \sigma_{cr}}\right)\left(1 - \mu_2\frac{a\,\sigma_{cr}}{\sigma_y - \sigma_{cr}}\right) \ldots\ldots\ldots (56)$$

[35] "Näherungsberechrung der Tragkraft exzentrisch gedrückter Stahlstäbe," by Karl Ježek, *Der Stahlbau*, Vol. 8, 1935, p. 89.

[36] "Die Festigheit von Druckstäben aus Stahl," by Karl Ježek, Julius Springer, Wien, 1937, p. 79.

[37] "Der Einfluss der Querschnittsform auf das Tragvermögen aussermittig gedrückter Bausthalstäbe," by Ernst Chwalla, *Der Stahlbau*, Vol. 8, 1935, p. 193.

[38] "Die Festigkeit aussermittig gedrückter Stahlstäbe beliebiger Querschnittsform," by Karl Ježek, *Der Bauingenieur*, Vol. 17, 1936, p. 306.

[39] "Die Festigheit von Druckstäben aus Stahl," by Karl Ježek, Julius Springer, Wien, 1937, p. 213.

Values of μ_1 and μ_2 for use in Eq. 56 are given in Table 1. Eq. 56 has unlimited validity for each of these sections except sections (d) and (e), for which it is valid, provided

$$\frac{\sigma_{cr}}{\sigma_y} \geqq \frac{S_c - S_t}{S_c + S_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

in which S_c and S_t are the section moduli at the compression and tension

TABLE 1.—VALUES OF μ FOR THE SOLUTION OF EQ. 56

μ	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
μ_1	0.5	0.5	0.5	0.8	0.8	0.4	0.4	0.9	0.9
μ_2	0.5	0.5	0.5	0.2	0.2	0.4	0.4	0.1	0.1

sides, respectively. If the relation defined by Eq. 57 does not exist, failure occurs at the tension side and the ultimate carrying capacity becomes

$$\left(\frac{l}{i}\right)^2 = \frac{\pi^2 E}{\sigma_{cr}}\left[1 - \mu_1 \frac{S_c\, a\, \sigma_{cr}}{S_t\,(\sigma_y + \sigma_{cr})}\right]\left[1 - \mu_2 \frac{S_c\, a\, \sigma_{cr}}{S_t\,(\sigma_y + \sigma_{cr})}\right] \ldots\ldots (58)$$

As the eccentricity ratio decreases (that is, as a approaches zero), the influence of the shape of the section diminishes.

It is possible to base column formulas on the ratio of the yield and critical stresses, or, for design purposes, on the ratio of a constant allowed stress and the critical stresses which correspond to the various $\frac{l}{i}$-ratios. In this case the force to which the column is subjected will be multiplied by this ratio (the dimensionless parameter $\omega \geqq 1$) and it is required that the resulting axial unit stress shall not exceed the constant allowed stress value. An exceedingly practical column formula of this type,[40] in the derivation of which the center deflection was not approximated by $y_m = \frac{c\,\sigma_E}{\sigma_E - \sigma_a}$ (which corresponds to the sine curve assumption of Eq. 54*a*) but by the more exact expression,[41]

$$y_m = \frac{c}{\sigma_E - \sigma_a}(\sigma_E - 0.25\,\sigma_a) \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

is as follows:

$$\omega = \tfrac{1}{2}\left[\varphi\left(\frac{l}{i}\right)^2 + a + 1\right]\left\{1 + \sqrt{1 - \frac{4\,\varphi\left(\frac{l}{i}\right)^2 (1 - 0.25\,a)}{\left[\varphi\left(\frac{l}{i}\right)^2 + a + 1\right]^2}}\right\} \ldots (60)$$

[40] "Vorschlage zur wirtschaftlichen Bemessung gedrückter Baustahlstäbe," by Ernst Chwalla, *Deutscher Stahlbau-Verband*, Berlin, Ernst, 1939, p. 18.

[41] "The Imperfect Strut and the Arch," by R. H. W. Hawken, *Proceedings*, World Eng. Cong., Tokyo, 1929, Vol. III, Pt. 1, pp. 73–114.

in which: $\sigma_E = \frac{\pi^2 E}{\left(\frac{l}{i}\right)^2}$; $\sigma_a = \frac{P}{A}$; and $\varphi = \frac{\sigma_y}{\pi^2 E}$. The critical stress becomes

$$\sigma_{cr} = \frac{\sigma_y}{\omega} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

Eq. 60 can be solved directly for any $\frac{l}{i}$-ratio; and for any variation of a it gives practically identical results with Eqs. 55.

All imperfections of the column in the structure are translated in column formulas to a single factor representing a certain equivalent eccentricity of the load. The functional relation between the eccentricity and the stress condition of the center section makes such a summary consideration of the imperfections possible. The practical validity of a column formula depends greatly on the assumption of the magnitude and variation of this measure of the degree of imperfection, expressed by the eccentricity ratio a.

The column formulas specified by the American Railway Engineering Association (A.R.E.A.; see Eq. 33*a*) and the American Association of State Highway Officials (A.A.S.H.O.) approximate the "secant formula," Eqs. 55 and 61 in which $a = 0.25$ $\left(\text{that is, } c = 0.25\,\frac{i^2}{e} \text{ has been assumed}\right)$. For the same symmetrical column section $\frac{i^2}{e}$ is constant and therefore the value of c remains constant for any l/i-ratio. This results in an increasing reduction of the calculated carrying capacity, or allowed unit stress, as the slenderness decreases. In actual structures $\frac{i^2}{e}$ is not constant; it increases with a decreasing l/i-ratio, and therefore the assumption of $a = 0.25 =$ constant results in increasingly more severe sectional requirements for compression members as the slenderness ratio decreases. The correctness of this statement will be demonstrated by an investigation of actual structures.

In Fig. 7 the $\frac{i^2}{e}$-values (or, for unsymmetrical sections, the average value) of the core radii $\frac{1}{2}\left(\frac{i^2}{e_t} + \frac{i^2}{e_c}\right)$ of main and bracing sections of several highway and railway bridges are plotted against the $\frac{l}{i}$-value of the members. These bridges have been built since 1920 and include long-span truss bridges, stiffening trusses of large suspension bridges, and other smaller structures. The indicated curve represents the average of scattering points. It appears from this curve that the variation of the actual $\frac{i^2}{e}$-values is hyperbolic in bridge structures, the product $\frac{i^2}{e} \times \frac{l}{i}$ being nearly constant. In addition to the ordinates $\frac{i^2}{e}$, Fig. 7 gives the values of c_t the unavoidable eccentricities, on

which the specification column formulas are based. The sensitivity of compression members to eccentric loads in the small $\frac{l}{i}$-region, in which rapidly increasing eccentricities are specified, indicates that, in bridge design, it is not reasonable to base a column formula on the assumption that $a = 0.25$ is a constant eccentricity ratio, because it results, unjustifiably, in an increasing factor of safety as $\frac{l}{i}$ decreases. A more uniform factor of safety will be obtained by assuming a linear increase of the unavoidable eccentricities c, with increasing $\frac{l}{i}$. In view of the hyperbolic variation of the actual $\frac{i^2}{e}$-values, this assumption results in an eccentricity

$$c = \kappa \tfrac{1}{2} \left(\frac{i^2}{e_t} + \frac{i^2}{e_c} \right) \left(\frac{l}{100\, i} \right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (62)$$

By Eq. 62 the design eccentricity becomes a function of the structural, sectional and dimensional characteristics of the compression member, and the expression of the eccentric ratio becomes:

$$a = \frac{\kappa}{2} \left(1 + \frac{e_c}{e_t} \right) \left(\frac{l}{100\, i} \right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

With present-day structural imperfections the value of the constant κ, for bridges and structures, is 0.25. The most unfavorable section actually in use is that composed of two angles, back to back. If the load produces compression in the extreme fibers of the legs in contact, the ratio of the distances from the center of gravity to the extreme compression and tension fibers is $e_c/e_t = 2.5$, for such sections. If the load is at the other side of the section e_t/e_c must be introduced into Eq. 63 and the yield stress in this case will occur at the tension side of the section. For symmetrical sections these ratios become unity, and for $\frac{l}{i} = 100$, Eq. 63 reduces to the present specification value of $a = 0.25$.

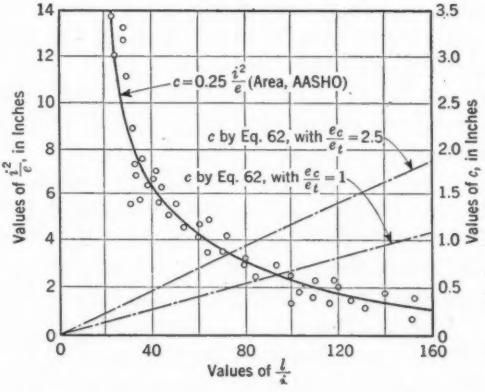


FIG. 7.—VARIATION OF THE RATIO $\frac{i^2}{e}$ WITH THE SLENDERNESS RATIO $\frac{l}{i}$ IN ACTUAL BRIDGES

Fig. 5 is an evaluation of Eqs. 54*a*, 54*b*, 55, 56, 60, and 61 for carbon steel and silicon steel. Curves 1 correspond to Eqs. 55 and 61

with $a = 0.25$; and curves 1a and 1b correspond to the same formulas with $a = \frac{0.25}{2}(1+1)\left(\frac{l}{100\,i}\right)^2$ and $a = \frac{0.25}{2}(1+2.5)\left(\frac{l}{100\,i}\right)^2$, respectively. Curves 2 correspond to Eq. 54*a* with $a = 0.25$. The critical stress, σ_{cr}, given by curves 1, 1a, 1b, and 2, is the axial, or average, stress at which the yield stress is reached in the extreme compression fiber, and $P_{cr} = A\,\sigma_{cr}$ is the lower limit, or design, carrying capacity of the column. Curves 3, Fig. 5, correspond to Eq. 54*b* and give the upper limit, or ultimate carrying capacity $P_{cr} = A\,\sigma_{cr}$ of a column with a rectangular section based on the classical theory of the yielding process and on the assumption that $a = 0.25$ is a constant eccentricity ratio.

The $\left(\omega = \frac{\sigma_y}{\sigma_{cr}}\right)$-curves are also given in Fig. 5. Examination of these curves reveals that they can be approximated by simple parabolic power relations, or, by means of these ω-curves, doubly curved cubic-hyperbolic $\left(\sigma_{cr} - \frac{l}{i}\right)$-curves, corresponding to eccentricity ratios which vary with $\frac{l}{i}$, can easily be represented.

Comparison of curves 1 with 1a and 1b in Fig. 5 indicates that, for a given value of E and σ_y, the magnitude and variation of the eccentric ratio a has a much greater effect on the values of the critical stresses and on the shape of the column curves than the other variables entering into the design of compression members.

In 1932 the Society's Special Committee on Steel Column Research concluded its 10-yr investigations with the statement that:[42] "A rational column formula can best be constructed on the basis of the theoretical formula for eccentrically loaded columns * * *." This statement is in agreement with the findings of numerous valuable investigations presented since 1926. Progress in the design of compression members, during the intervening years, is characterized by the adoption of the theoretical formula as evidenced also by both the A.R.E.A. and A.A.S.H.O. specifications which have been in force since before 1935. Improvements on the A.R.E.A. formulas and the A.A.S.H.O. formulas so that they will conform to actual structural conditions of eccentricity and the use of proper reduced lengths, constitute the criteria for their usefulness, rather than the formal requirements suggested by Mr. Osgood.

The determination of the reduced length should be based on consistent assumptions. Inasmuch as the design of the members is based on the condition of yield stress in the extreme fiber it is reasonable to consider the reduced lengths which correspond to this condition of the structure. This indicates that such summary assumptions for the buckling lengths as those made by Mr. Osgood and those involved in the A.R.E.A. and A.A.S.H.O. column formulas are neither reasonable nor tenable. The variation of the stress and rigidity of members adjacent to the same panel point, the character of the intermediate supports of members in various truss systems, and other factors in-

[42] "Final Report of the Special Committee on Steel Column Research," *Transactions*, Am. Soc. C. E., Vol. 98 (1933), Conclusion 5, p. 1460.

fluence the buckling length. For all members and conditions the specification formulas assume 0.75 l if the connections are riveted. This fact, combined with the assumed constant eccentricity ratio, makes the design value of these fundamentally correct formulas no greater than rules of thumb.

The critical stresses represented by curves 1a and 1b in Fig. 5—in combination with reduced lengths obtained from the yield stress in the extreme-fiber condition of the members in the structure and correspondence between the allowed stress in tension and compression—are more likely to result in a homogeneous factor of safety at economic advantage than the prevailing specification column formulas which are not fitted to the characteristics of actual structures.

The writer has attempted to explain in the foregoing why there is no need for the application of the double-modulus theory in structural design. The difference between the Engesser-Kármán (double-modulus) line and the line of the ideal buckling stresses is greatest where the Euler hyperbola intersects the horizontal of σ_y (Fig. 6). Assuming this difference to be as great as 10% of σ_y, introducing this value into Eq. 54a, and solving, $a = 0.01$. Inasmuch as the design value of a for actual members in this region is between 0.25 and 0.6, the carrying capacity resulting from the assumption of the ideal stress-strain diagram is only 1.5% to 4% different from that obtained by the use of the double-modulus line in the extreme case.

The problem in design is to ascertain a desired structural performance with the greatest economy. The comparison of various structural schemes is equitable only if their factors of safety are the same. Empirical equations, and the prevailing specification formulas, do not result in useful comparisons because their deviations from structural reality affect members of different proportions in different ways. It can generally be stated that the most frequent stress condition—compression combined with bending—is not being given proper attention in design. Laborious calculations, by which internal forces and moments are often determined, usually end in taking the summation of the divisions of force by area and moment by section modulus. The result is not indicative for the factor of safety of the members unless the critical stress that corresponds to the loading condition is known.

The writer agrees with the statements made by Mr. Osgood in the "Summary." There is no point to approximating a correct formula by substituting another of limited validity. It is practical to prepare tables computed from a formula which will apply correctly to the case under consideration in design specifications. Such a method, however, is not applicable directly to many of the infinitely varying complex problems of engineering practice. The simplification of such problems to manageable form requires judgment ripened by thorough structural knowledge, and it requires the study of analytic and experimental investigations in the entire field of stability phenomena.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

INTERRELATION OF CERTAIN STRUCTURAL CONCEPTS

Discussion

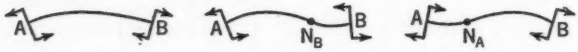
BY DEAN F. PETERSON, JR., AND A. A. EREMIN

DEAN F. PETERSON, JR.,[32] ASSOC. M. AM. SOC. C. E.[32a]—Using the slope-deflection equations, Mr. Weiss has developed the algebraic interrelationships between the various "structural concepts" commonly used in the method of moment distribution. The "common factor" used to derive the equations and to illustrate the interrelationship is the concept of the "fixed point."

In the "Introduction," Mr. Weiss states:

> "Usually such points [points of zero bending moment] are thought of in connection with beams continuous over three or more supports; but two such 'fixed points' exist in every member of any frame. They are called 'fixed' because their position on the elastic curve is stationary."

The position of the "fixed point" is actually stationary in only one direction since it is free to deflect transversely. The choice of the term "nodal" further implies that this condition is not true. The number of "fixed points" occurring at any time depends upon the moments acting on the member. Either none, or one, will actively occur in each of the unloaded spans (Fig. 11). When distributing a single fixed-end moment to a frame, all the unloaded members will have one and only one fixed point, which will occur either at N_A or N_B. The distance measured along the span to this point always will be the same regardless of the magnitude or sense of the moment.



(a) NO FIXED POINT (b) ONE FIXED POINT (c) ONE FIXED POINT

FIG. 11

NOTE.—This paper by Camillo Weiss was published in January, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1945, by Ralph W. Stewart, and R. C. Brumfield; June, 1945, by Max W. Strauss; September, 1945, by Leon Beskin, Alexander Dodge, and William C. Spiker; and October, 1945, by T. F. Hickerson.

[32] Melbourne Beach, Fla.

[32a] Received by the Secretary October 8, 1945.

The solution of Eq. 9 for n_{AB} in the frame of Fig. 1 requires the use of n_{BC} which is unknown since it depends on the restraining properties of joint C. Similarly, the restraining properties of joint C depend upon those of joint D, etc. Presumably it is mathematically necessary to start the solution at some joint where the restraining properties at the far ends of all of the concurrent members, except one, are known. Having solved for n at such a joint, it is possible to solve the equations at the other joints progressively. If no such joint exists the equations must be written for each joint and solved simultaneously. This is merely a solution of the slope-deflection equations for the entire structure with an applied moment at A in terms of the parameter n and the amount of computation involved is about the same as if the conventional equations had been used.

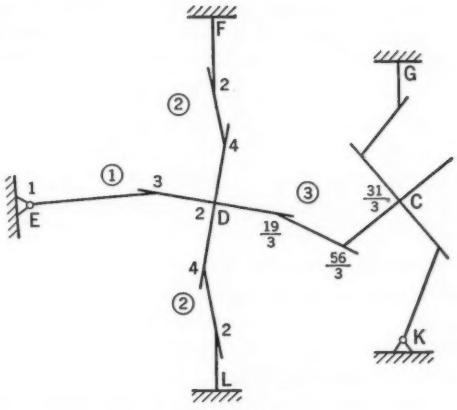


FIG. 12

The solution of n may be made directly from the geometry of flexure by using an elastic curve traverse.[33] For example, in Fig. 12, relative stiffness factors have been assigned to a part of the frame of Fig. 1 and joints F and L are assumed to be fully restrained. To find n_{CD}, assume that a bending moment induced from joint C acts to rotate joint D. Draw the traverse. Since the traverse angles marked 19/3 and 56/3 are a measure of the moments existing at joints D and C, respectively, $n_{cD} = \dfrac{\frac{56}{3}}{\frac{19}{3}+\frac{56}{3}} = \dfrac{56}{75}$. The traverse may be continued for the entire frame and the fixed points for each member can be found as easily, if desired.

However, moments might just as well have been determined in the first place. As an example, suppose the moment to be carried over from end C is 100; then all of the moments in the members about joint D may be computed immediately since they are proportional to the traverse angle and the relative stiffness: $M_{DC} = \dfrac{100}{\frac{56}{3}} \times \dfrac{19}{3} = 34.0$; and $M_{DF} = \dfrac{100}{\frac{56}{3}} \times 4 \times \dfrac{3}{2} = 14.3$. Similarly: $M_{FD} = 7.2$; $M_{DE} = 5.4$; $M_{DL} = 14.3$; and $M_{LD} = 7.2$. The traverse could easily be extended through the frame to point A and all moments due to the moment at A computed.

[33] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521.

In the "Introduction" Mr. Weiss states:

> "The method of moment distribution introduced by Hardy Cross,[2] M. Am. Soc. C. E., offers a relatively rapid and precise means of computation and, in the writer's opinion, is as good a method as can be devised. It permits the analysis of a framework from a unique point of view. Whereas the method of least work is based on the concepts of work and energy; the slope-deflection method on relations governing deformations comprising the geometry of flexure; the Cross method may be considered as based on a complex of concepts that can be summed up under the term 'restraint.'"

It is the writer's desire to point out that the concepts summed up under the term "restraint" are basically associated with the relations comprising the geometry of flexure. The solution for n, Fig. 12, is based on an extension of these relationships. It is further noted that the slope-deflection equations (Eqs. 2) are the basic equations of the author's mathematical development.

From a consideration of Eqs. 9, 10, 11, and 12, it may be stated that the "restraint" imposed by any joint, not fully fixed or hinged, in a continuous structure is dependent upon every member and upon every other joint in the frame. Prior to the introduction of the Cross method this complete interdependency was stated in the form of a set of simultaneous equations involving, as unknowns, rotations or moments at each joint. In order to avoid the usually arduous task of solving these equations, Professor Cross developed a means of "distributing" unbalanced moments to the concurrent members of a joint assuming the far ends of the members to be fixed. By successively releasing each joint the effects of the "restraint" of remote joints and members were added as increments to the moment at the original joint. In this method the conceptions of "stiffness factor," "carry-over factor," and "distribution factor" are based on the assumption that the remote ends of the members are fully fixed. In Mr. Weiss' paper these factors are based on the true "restraint" of the joints at the far ends. Therefore, in the general case, none of them can be evaluated except after, or in connection with, some kind of a complete analysis for a moment imposed at some joint on the frame. Since such an analysis will also determine the resulting moments (which is the information that interests the designer), the various concepts set forth by this paper, although interesting from an academic point of view, appear to be of secondary importance in practice. For example, while, from the solution of the frame of Fig. 12, it is possible to determine n_{CD}, the moments resulting from the loading were also determined by the same analysis. This would generally be true regardless of the method of analysis. The use of the terms "stiffness factor," "carry-over factor," and "distribution factor" to describe physical conceptions somewhat different than those which they describe in the method of moment distribution may lead to some confusion.

A. A. EREMIN,[34] Assoc. M. AM. SOC. C. E.[34a]—It is true, as Mr. Weiss has stated, that a uniform mathematical language, composed of commonly rec-

[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[34] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[34a] Received by the Secretary October 9, 1945.

ognized terms, simplifies the presentation and development of computation methods. This is true regardless of how often such "standard" symbols may need to be revised. As knowledge of a subject increases and new symbols are introduced, the alphabet is often insufficient to cover the required terms. Furthermore, normal progress will require new interpretations of the physical properties of a structure. For instance, the term, "fixed point," is not defined conclusively at present. This term may be applied to three pairs of fixed points—central points, inflection points, and elastic points, as explained by the writer elsewhere.[35] The position of all these characteristic points is controlled by the physical and geometric properties of structures. With doubtful justification, Mr. Weiss introduces the term "node point" to denote an inflection point. The term "inflection point" is more directly associated with the characteristic variations in the shape of a member, its neutral axis, and moment diagram. Less effort is required to memorize the various steps in the moment-distribution method when thought of in terms of inflection points.

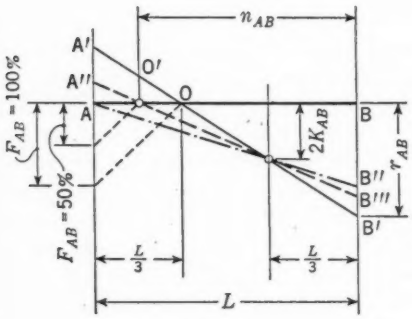


FIG. 13

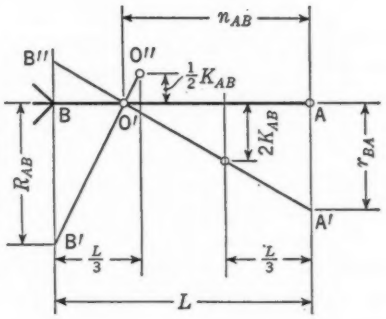


FIG. 14

By a simple change in the notation the authors' algebraic expressions for the restraining factors may be converted to those derived by the writer in 1943.[36] These restraining factors also may be obtained by simple graphical methods, resulting in a saving of time and effort. The member AB in Fig. 13 has a constant section, its basic stiffness being $K_{AB} = I/L$. The stiffness factor of AB at B, with end A fixed, may be determined as follows: Draw line A'B' through the third point of the span and through the ordinate 2 K_{AB} at the third point of the span near end B. The ordinate BB' is equivalent to the stiffness factor r_{AB}. If the restraint at end A is reduced to zero by introducing a hinge the stiffness line will be revolved about the ordinate 2 K_{AB} to the hinge at end A. The ordinate BB'' is equivalent to the stiffness factor of AB with the opposite end hinged. If the restraint at end A is reduced to one half of the full restraint, the inflection point in this case will occur midway between end A and the third point of the span, and the stiffness line will be revolved about the ordinate

[35] *Proceedings*, Am. Soc. C. E., November, 1944, p. 1490.

[36] "Analysis of Continuous Frames by Graphical Distribution of Moments," by A. A. Eremin, published by the author, Sacramento, Calif., 1943.

2 K_{AB} to the inflection point. The ordinate BB‴ is equivalent to the stiffness factor of AB with the opposite end 50% restrained. The dashed parallel lines near end A in Fig. 13 illustrate the graphical construction of the inflection-point location according to the given degree of restraint.

The graphic construction for locating the inflection point in a rigid frame is shown in Fig. 14. Member BA in Fig. 14 is the same as that shown in Fig. 3. The inflection point O′ is determined by intersecting the member axis BA with line BO″ drawn through the ordinate of the restraint R_{AB} at end B and the ordinate 1/2 K_{AB} at the third point of the span. The stiffness factor r_{BA} is determined as shown in Fig. 13.

The inflection points are used with a considerable advantage in the graphical distribution of moments. Therefore, it is gratifying that Mr. Weiss has adopted the inflection points also in his algebraic formulas. Obviously the author's restraining factors will facilitate development of various semigraphical methods of moment distribution.

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DISCUSSIONS

DESIGN OF ROADBEDS

Discussion

BY JACOB FELD, AND JASPER L. STUCKEY

JACOB FELD,[24] M. AM. SOC. C. E.[24a]—The paper analyzes the stability of the soil as a mixture of gas (air), liquid (water), and soil solids (rock fragments), and attempts to fix limiting percentages of each state of matter as a criterion for the absence of undesired physical characteristics. Actually, the argument presented is applicable to all soil stability problems, not only to roadbeds.

Complete logical solutions of the problem can be expected only when all the factors are considered. A soil is not merely a mixture of solid rock fragments, liquid water, and gaseous air. The air fraction may be partly gas, partly liquid, and even partly solid (occluded gas held as solid constituents by certain minerals and metals). The water in the soil may be (and usually is) partly gas, partly liquid, and partly solid. The rock fragments are sometimes entirely solid (clean granular soils), but also sometimes entirely liquid, and even partly gaseous (dust storm and soot additions). The state in which the soil constituents occur depends largely upon the absence or presence of colloid admixtures or impurities. The colloids may result from the mechanical or chemical disintegration of the rock fragments, or from animal and vegetable refuse leachings. Minute colloid admixtures, if chemically and electrostatically correct, have major control of the physical action of soil mixtures.

When the influence of colloids is fully appreciated, and when a soil can be properly identified and analyzed for all its physical and chemical constituents, not only can the physical behavior of the soil be prophesied, but the desired physical characteristics can be produced by the addition of colloidal admixtures.

In the 1921, 1922, and 1923 progress reports of the Special Committee to Codify Present Practice on the Bearing Value of Soils for Foundations,[25] there is included an exhaustive treatment of the colloidal properties of clay as a soil constituent. In a discussion of these reports, the writer extended that treat-

NOTE.—This paper by Ira B. Mullis was published in April, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1945, by Charles W. Britzius, J. T. L. McNew, and Gustavo Pérez Guerra.

[24] Cons. Engr., New York, N. Y.

[24a] Received by the Secretary August 27, 1945.

[25] *Proceedings*, Am. Soc. C. E., February, 1921, p. 9; March, 1922, p. 523; and October, 1923, p. 1732.

ment to include the other colloids often found in soils—silicic acid, silicates, aluminum and iron hydroxides, humus acid, as well as colloidal clays of various types.[26] The effect of various colloids on different soil types is there described not only on the physical property but also on the fertility of soil. Much useful information on this subject can be found in the reports of soil studies by the agronomists.

That air, water, and rock fragments are not always gas, liquid, and solid, respectively, at normal temperatures and pressures, is not a new idea. For instance, Galileo, in 1640, understood that there was a difference between liquid water and pore water. He states that the smallest parts into which water can be divided must be smaller than the smallest consistency of water; for, no matter how small or finely a powder is divided, the particles can still be wetted. That water is different from the smallest divisible quantities of liquid water.[27]

In 1920, Karl Terzaghi, M. Am. Soc. C. E., stated that water films less than two millionths of an inch in thickness are semisolid in action. Such water does not boil or freeze at normal temperatures. C. A. Hogentogler,[28] Assoc. M. Am. Soc. C. E., has shown that, although the division between the solid state and the plastic state depends on the percentage of moisture, the percentage decreases as the soil is put under pressure; and the pressure can be applied either externally or internally, the latter by reduction of the water film thicknesses. H. F. Winterkorn,[29] Assoc. M. Am. Soc. C. E., has added, to this study, the proof that the entire relation between moisture percentage and plastic state can be changed by adding chemicals to the water.

When it is kept in mind that, as W. D. Bancroft[30] states, all solids tend to adsorb or condense upon their surfaces any gases or liquids with which they come into contact, the importance of colloid-sized particles must be evident; for colloidal particles have very great ratios of surface area to weight.

Many practical applications of soil control by colloids and of clay soil control by chemicals can be found in recent reports. One application developed by experience, and not as a result of laboratory or analytical study, may be of interest. Seneca County, in New York State, is almost entirely covered by soft shale and extremely fine clay. When the surface is disturbed, there are only two conditions—dust clouds when dry and a slick slippery surface when wet. Nevertheless, local farmers have used a layer of crushed local shale rock, taken from the exposures where thin limestone seams occur, to build the secondary roads. The rock, in pieces up to 50 lb in size, are spread along the desired location, and preferably they are spread directly upon the ground surface. Traffic and weather form a fairly smooth road surface in a few months. These surfaces, locally known as "shuck roads," cannot be made from shale that lacks the requisite limestone content.

When the writer was required to produce large areas of serviceable parking

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[26] *Proceedings*, Am. Soc. C. E., January, 1924, p. 114.
[27] "Opere di G. Galilei," by Galileo Galilei, 1808 Ed., Vol. 3, p. 25.
[28] *Proceedings*, Highway Research Board, Vol. 18, Pt. II, 1938, p. 9.
[29] *Ibid.*, Vol. 21, 1941, p. 415; also Vol. 16, 1936, p. 293.
[30] "Colloid Chemistry," by W. D. Bancroft, McGraw-Hill Book Co., Inc., New York, N. Y., 1932.

fields and bus terminal pavements, with a minimum of labor and no outside materials, as part of the construction facilities for a war project in Seneca County, he applied the "shuck road" idea, with very good results. A local fine sand deposit was found to contain a large limestone content, from the leaching of limestone bedrock. The sand was unfit for construction purposes. By disking about 1 in. of this sand into the clay surface, adding a slight quantity of water, and rolling with a 10-ton roller, a hard impervious surface was obtained. The top crust, about 4 in. thick, hardened in a few days and was used as a parking field, with practically no maintenance, for a year. A sprinkling of limestone chips was applied in the bus terminal areas, and these were used for three years with little maintenance.

The paper by Mr. Mullis is of great value in pointing to a logical approach to soil compaction studies. The method can be made of far greater value by adding the necessary factors of colloid action.

JASPER L. STUCKEY,[31] ESQ.[31a]—Strong arguments are presented by Mr. Mullis for the rigid control of liquids and solids in the development of stable roadbeds; also excellent reasons are offered for a better knowledge of geological materials and the fundamentals of geology on the part of the highway engineer. A better appreciation of both of these should be helpful to all highway engineers, and especially to those concerned with the problem of constructing permanent roadbeds for modern traffic conditions.

Long before the development of modern roadbeds it was known that too much water on the roadway caused softness and ruts. As a result, roads generally followed the ridges and divides in order to take advantage of natural drainage. It is but natural that, with changed demands for roads and highly specialized facilities for research, methods should be developed for producing permanent roadbeds where they are needed. Mr. Mullis should be especially commended for the contribution he is making to this end.

It is interesting to note that of fourteen topics offered as a guide to any discussion of the paper, seven deal with the physical properties of the geological materials necessary in roadbed construction. It is also interesting that these materials are properly designated and not erroneously called "soil" as is so often done.

Since most of the materials used in roadbed construction are obtained along the line of the road or from local sources of supply, the engineer should consider two factors: (1) The geological conditions that may affect the permanence and stability of the roadbed; and (2) the kind, character, and proper utilization of the materials available for construction. Failure to do this has caused, and is causing, entirely too many roadbeds to prove unsatisfactory.

Dense rocks such as granites, quartzites, and many limestones, in undisturbed masses, might be considered ideal materials for roadbed foundations. They have low porosity and are known to be highly resistant under most conditions. In the majority of cases, however, fresh, unaltered rock of any type is seldom found in adequate quantities at or near the surface. In many areas,

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[31a] Received by the Secretary August 31, 1945.

weathering and rock decay have produced a mantle, of variable thickness, of decayed rock material which overlies the hard and fresh rock and grades gradually into it. If, as in the Atlantic Coastal Plain, sediments which are not deeply weathered are found at or near the surface, they are often of variable character and poorly consolidated. Since this is true, the highway engineer is forced in most cases to use materials that have high porosity and low resistance and require special treatment to be made into a stable mass such as is required in a modern roadbed.

Building stone usually is tested dry because its strength often decreases when wet. T. L. Watson, F. B. Laney, and G. P. Merrill[32] have shown that sandstone, with an absorption of 3.71% and 4.2%, tested only slightly more than one half as strong when saturated as when dry. This evidence strongly supports the argument for the controlled mixing of fluids with solids if roadbeds are to be stabilized.

The crushed stone, gravel, and sand used in the production of concrete for road paving and the preparation of the concrete have been standardized and closely controlled for many years. This paper definitely indicates the need for higher standards and better controls in the building of roadbeds.

[32] *Bulletin No. 2*, N. C. Geological Survey, 1906, pp. 235–236.

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DISCUSSIONS

SEDIMENTATION AND THE DESIGN OF SETTLING TANKS

Discussion

BY P. CHARLES STEIN, AND LYNN PERRY

P. CHARLES STEIN,[44] ASSOC. M. AM. SOC. C. E.[44a]—It is obvious that a heavy particle suspended in a liquid will tend to settle. The difficulty encountered in applying this casual concept of settling to the design and operation of sedimentation units furnishes enough proof of the need for understanding the principles of settling and related phenomena. A technical basis for the design of settling processes must depend upon specific knowledge of the properties of the suspensions to be treated and of the physical phenomena associated with flow through tanks.

This paper should make more engineers aware of the mechanics of settling tanks. Its scope extends beyond that mission, however, in that the concepts involved in the more important parts of the problem are developed and definite design principles are formulated.

It is to be hoped that the paper will help dispel the erroneous notion that detention time is the important factor in the removal of suspended matter. For a given settling velocity both depth and detention time govern degree of removal since, with a deeper tank, a particle will require a longer time to reach the bottom. Thus, with a given settling velocity, the ratio of depth to detention time determines the number of particles reaching the bottom of an ideal tank. Simple algebra will show that this ratio is equal to the rate of flow divided by the surface area of the tank, which is the definition of overflow rate given by the author following Eq. 6. When one considers a particle settling at a given rate in a tank, it is simple to grasp the fact that depth divided by settling time is the proper criterion for removal and that neither depth nor time alone has real significance.

The tests mentioned by the author are not simple routine ones because standardized methods have not been formulated. Thus far, data on physical

NOTE.—This paper by Thomas R. Camp was published in April, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1945, by Norval E. Anderson, and R. A. Mulholland.

[44] Cons. Engr. (Kuchler & Stein), Sharon Hill, Pa.

[44a] Received by the Secretary August 31, 1945.

properties of suspensions encountered in water and sewage treatment are extremely meager, as the author states. This scarcity of information means not only that there are no acceptable normal values at present to serve as comparators but also that little has been done in the way of developing standard techniques. This lack of development does not nullify the value of the method of design described by the author. Instead, realization of the shortcoming should spur sanitary engineers toward a more scientific study of the treatment methods that they use.

For new units it is necessary, before design, to determine the nature of the suspension that will be obtained when the unit is in operation. If the new unit is an addition to an existing plant, design information may be secured by suitable interpretation of tests conducted on suspensions in existing units. If the unit is to be a part of an entirely new plant, then recourse should be had to sanitary engineering laboratory design methods or to a pilot plant. In the laboratory method a proposed process is examined critically to determine the variables entering into each stage of the process. Then, the process is completed step by step on batch samples, the important variables in each step such as chemical characteristics, mixing time, degree of turbulence, velocity gradient, etc., being controlled with a view toward determining the optimum values of the several variables. In the course of this procedure, the settling velocity analysis can be made at the proper time in the treatment routine. It is assumed that the steps prior to sedimentation will have been finished in a container of adequate design and dimensions so that there will be no necessity for transferring the sample to another apparatus for the settling analysis.

The paper exposes the neglect with which investigators have treated the mechanism of flocculation and its relationship to the settling problem. Eq. 26, which was introduced by M. von Smoluchowski,[45] and Eqs. 25 are based on rather crude assumptions, since their purpose in this paper is to show relative rather than absolute effects. In their development, the actual hydrodynamics of fluid flow about the particles has been ignored. Microscopic studies by the writer indicate that the process of flocculation by settling is much more complex than the simple mechanism pictured by the assumptions leading to Eqs. 25. Similar comments apply to Eq. 26 but analysis of the results of Darrell A. Root[31] along with torsion dynamometer measurements by the writer to determine work input on the Root apparatus indicate that the proportionalities between rate of flocculation and concentration and between rate of flocculation and velocity gradient are valid.

Despite the very approximate nature of Eq. 26, it discloses important relationships concerning the effect of flocculation during settling. Although actual problems involve polydisperse suspensions, which have been extensively studied by the writer, for simplicity, the considerations will be limited to particles of uniform size. The conclusions with such a restriction are qualitatively valid.

For spherical particles, it is evident that the number of particles per unit

[45] "Versuch einer mathematischen Theorie der Koagulations-kinetic kolloider Losungen," by M. von Smoluchowski, *Zeitschrift für physikalische Chemie*, Vol. 92 (1917), p. 155.

[31] "The Effect of Temperature Upon the Rate of Floc Formation," by Darrell A. Root, thesis submitted to Mass. Inst. Tech., Cambridge, Mass., in 1939, in partial fulfilment of the requirements for the degree of Master of Science.

volume of the suspension will equal the volumetric concentration divided by the cube of the diameter times a constant. Introduction of this relationship into Eq. 26 discloses that the number of contacts per unit volume per unit of time is proportional to the square of the concentration, and that the percentage of particles involved in contacts is proportional to the volumetric concentration and is independent of the size of the particles. For a velocity gradient of 10 sec^{-1} and a volumetric concentration of 2 parts in 10,000 (roughly equivalent to 200 ppm suspended solids in sewage), the initial flocculation rate given by Eq. 26 is equivalent to contacts involving 12% of the particles per minute. For lower or higher concentrations, the rate of flocculation will be proportional.

From the foregoing, it should be clear that the effect of flocculation on removal should be particularly pronounced where the volumetric concentration of suspended matter is high. With the volumetric concentrations present in water and sewage treatment processes, the effect of flocculation on settling can be substantial. It is only with low concentrations of the order of 10 ppm or 20 ppm that the flocculation effect would be negligible unless both velocity gradient and detention time are large.

The application of the meager available information concerning flocculation in settling tanks certainly indicates that the shallow tanks dictated by the general settling theory are also more favorable from the standpoint of flocculation during settling. It will remain for pilot plant testing to demonstrate the advantages of the method of design presented by the author. Testing should be done also in units of existing plants. The results might well indicate that a necessary increase in settling capacity can be attained by inserting horizontal diaphragms in existing tanks at considerably less cost than would be required for building additional tanks.

The writer would like to point out what he believes to be an error in Example 2, since it might lead to faulty design of final settling tanks for activated sludge. In this example, it is assumed that the concentration of the sludge in the sludge blanket is a function of the time of compacting only and is independent of the depth of the blanket. This means that, for a given degree of compaction, the rate of subsidence of the sludge blanket is proportional to the thickness of the blanket. That this cannot be a valid conclusion is indicated by a consideration of settling and compaction in a very deep tank. The assumption that rate of compaction, in terms of the ratio of the volume occupied by the sludge blanket, is independent of depth will lead to the conclusion that in a very deep tank the sludge blanket must sink at a velocity greater than the hindered settling velocity. This, obviously, is not possible. If, in Example 2, for a tank 10 ft deep, the position of the top of the blanket for a sludge concentration of 6,000 ppm were calculated, it would be found that the rate of subsidence from point A of Fig. 4 to the depth corresponding to 6,000 ppm would be about 0.14 cm per sec, whereas the hindered settling velocity is only 0.11 cm per sec.

There is a similarity between the problem of compaction of sludge and of the consolidation of clays as found in soil mechanics problems. In both cases, the rate of consolidation in a lamina is proportional to the normal gradient of the hydrostatic excess pressure. In a sludge blanket at a given degree of consolidation, it may be argued that the hydrostatic excess pressure at any point

is roughly proportional to the depth of sludge over that point. Thus, the pressure gradient causing consolidation will be roughly independent of the depth in the early stages of the compaction process. This means that the rate of drainage of interstitial fluid will be independent of depth and that two blankets, one shallow and the other deep, would compact with approximately the same absolute velocity of subsidence. In the final stages of consolidation the compaction in deep tanks will be greater than in shallow tanks because of the increased weight of sludge over the bottom layers.

In general, the results reported by Willem Rudolfs, M. Am. Soc. C. E., and I. O. Lacy,[16] cited by the author, are at variance with the author's conclusions respecting the effect of depth on sludge compaction. To secure data on this specific problem, Messrs. Rudolfs and Lacy made tests of settling and consolidation in glass tubes with initial depths of 117 in., 93 in., and 45 in. The results taken from curves[16] presented by Messrs. Rudolfs and Lacy have been calculated to give a nondimensional plot, devised by the writer. In this plot, presented in Fig. 19(*a*), a value of $v = 2.5$ in. per min was used in com-

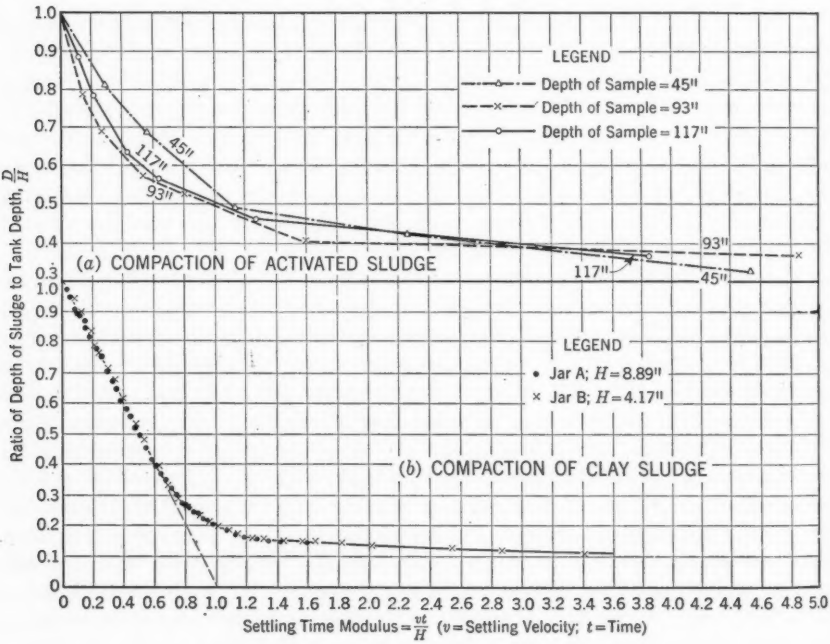
LEGEND
Depth of Sample = 45"
Depth of Sample = 93"
Depth of Sample = 117"
45"
117"
93"
(a) COMPACTION OF ACTIVATED SLUDGE
LEGEND
Jar A; H = 8.89"
Jar B; H = 4.17"
(b) COMPACTION OF CLAY SLUDGE
Ratio of Depth of Sludge to Tank Depth, D/H
Settling Time Modulus = vt/H (v = Settling Velocity; t = Time)

FIG. 19.—EFFECT OF DEPTH ON RATE OF COMPACTION

puting the abscissa. If the settling velocity were the same for the three samples and the absolute rates of subsidence were equal, the curves for the three depths would coincide. (In Fig. 19 the settling time modulus $\frac{vt}{H}$ is

[16] "Settling and Compacting of Activated Sludge," by Willem Rudolfs and I. O. Lacy, *Sewage Works Journal*, Vol. VI, no. 4, 1934, p. 647.

plotted against the ratios and D/H as ordinates, definition being as follows: v = settling velocity; t = time; H = tank depth; and D = depth of sludge.

Actually, the plotted points show a substantial scattering. However, it will be observed that the placement of the curves does not represent a consistent order, the results from the deepest tube being between those of the intermediate and lowest depth. After 19 hr of settling the results were consistent, the sludge depths being, respectively, 12%, 14%, and 16% of the liquid depths for the 117-in., 93-in., and 45-in. samples. The writer believes that the tests shown in Fig. 19(*a*) constitute a rough check of his contention that the absolute rate of consolidation in the early stages is independent of sludge depths. The failure to achieve perfect coincidence of the curves is due to experimental error. From the breaks in the curves for the individual samples, the writer would conclude that there were arching effects present due to the use of settling tubes which did not have a sufficient diameter. The writer has found, in consolidation tests made with calcium carbonate and with clay suspensions, that the settling tube must be large with respect to the depth of the sludge if arching effects are to be negligible.

To secure data of greater precision, the writer performed consolidation tests with two jars containing a floc made by coagulating a suspension of red clay with ferric chloride, ammonia, and sodium chloride. Both jars were 5.2 in. in internal diameter. Jar A had an initial depth of 8.89 in. and the depth in jar B was 4.17 in. After coagulation, the floc was poured back and forth in the two jars to insure equality in condition. It was found that the hindered settling velocities in the two jars, as well as the time rate of subsidence at equal degrees of compaction, were identical within the limits of experimental error as shown on the nondimensional plot presented in Fig. 19(*b*). The hindered settling velocity of the floc used was found to be 0.125 in. per min and the total duration of the test was 9 hr. Although only the first quarter of the test is shown in Fig. 19(*b*), the results of the two samples gave identical curves even for the end of the test. It is to be expected that there would be some disparity between the two samples eventually because of the larger final intergranular pressure in the sludge blanket for the deeper sample.

This material is presented because it serves to throw a small amount of light on the problem of sludge compaction, which is a part of final settling tank design. The data presented were obtained with a clay floc and not with floc from activated sludge plants. Unless the floc from activated sludge plants is radically different from other types of materials, the general conclusions of the test should be valid. Comparison of Fig. 19(*a*) with Fig. 19(*b*) shows that the general types of curves obtained with activated sludge and with a clay floc are the same.

It would appear, then, that better compaction in a given detention time would be possible with shallow final settling tanks than with deep tanks; or, for equal degrees of compaction, less time is required with thin sludge blankets than with thick ones. Thus, shallow final settling tanks would require less detention time to obtain a specified concentration of sludge. With activated sludge, this is an important biological consideration because long absence from a medium rich in oxygen will reduce the viability of the active organisms.

LYNN PERRY,[46] M. AM. SOC. C. E.[46a]—Concepts of the fundamental principles of flow and fluids have changed radically during the past generation. Capable investigators with laboratory equipment, frequently inadequate, have correlated widely scattered bits of information, checked over the fundamentals, and secured an immense amount of supporting laboratory data. This work is by no means complete.

The more mature men of the engineering profession who were brought up on "Merriman" and "Hughes and Safford," with their detail about Kutter and other early investigators, find it extremely difficult to imagine that the film of a moving fluid in contact with the wall of a conduit is stationary. They have been engaged in design and construction during the most active and progressive generation in the history of the human race; and they will hold to the empirical methods of design which have been tried and found fairly satisfactory for their purpose. It is the younger generation who, with proper training, will base their fluid analyses on rational fundamentals.

Dimensional analysis gives the proper form for an equation but, unfortunately, it does not yield a numerical result. Numerical results, sufficiently accurate for design, are required by the designer, and are required promptly. Usually, theoretical analysis must be supplemented by considerable laboratory work before it can be put into form for the designer to use. Unfortunately, most of the data that have been made and published in support of empirical formulas are not exactly what is required.

Designing engineers are constantly facing conditions where a deposition of solids is necessary, where no deposition is permitted, or where a partial deposition is desired to separate heavier suspended solids from the lighter. Some years ago, the writer came into the possession of information which led to the curve presented in Fig. 20, the equation of which is

$$V = 424\ (G_r - 1)\ D^2 \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

In Eq. 38, V is the velocity of water in which sand of various particle sizes will settle; G_r is the specific gravity; and D is the diameter of a particle. This curve has been used and found to be trustworthy. The line is for sand—not spheres, cubes, or any other geometrical figures—plain sand ($G_r = 2.35$). It has been checked from time to time, and the small circles indicate the results of laboratory observations at Lafayette College, Easton, Pa. For heavier or lighter suspended solids other such curves are available. For velocities and sizes below the curve, deposition will occur; for velocities above the line, the solids will be carried along with the current of water. In many industrial processes the velocity of the water carrying the solids in suspension can be controlled. The necessary controls for maintaining a constant velocity of water or sewage passing through a municipal treatment plant would be a subject for another paper. Engineers are competent to design a satisfactory water or sewage treatment plant for a predetermined rate, and for a specific quantity and type of suspended and dissolved solids which require a given type, quantity, and quality of chemical. Few designers ever become the operators of the

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[46a] Received by the Secretary October 4, 1945.

plants they have designed and built. Although they have an academic knowledge of operation, they seldom face the trying problems that confront the operator. This trend in engineering development has produced a rift between the designers and the operators. An unequal settlement of a few hundredths of a foot is the source of much more operating anxiety than a failure to secure a slightly higher efficiency in sedimentation. The designer is always seeking perfection even though he knows that goal is unlikely. He deals almost entirely with the forces of nature. The operator, on the other hand, is constantly confronted with the frailties of human nature.

Theoretically, engineers should be able to develop the desired effect in the laboratory or in the pilot plant, and attain the same effect in the prototype. When this cannot be done, it indicates that there is something about the model law that is not quite familiar, and another set of empirical constants must be developed to give fairly accurate numerical results. Either engineers should be able to compute these constants or their dimensional analysis needs a review.

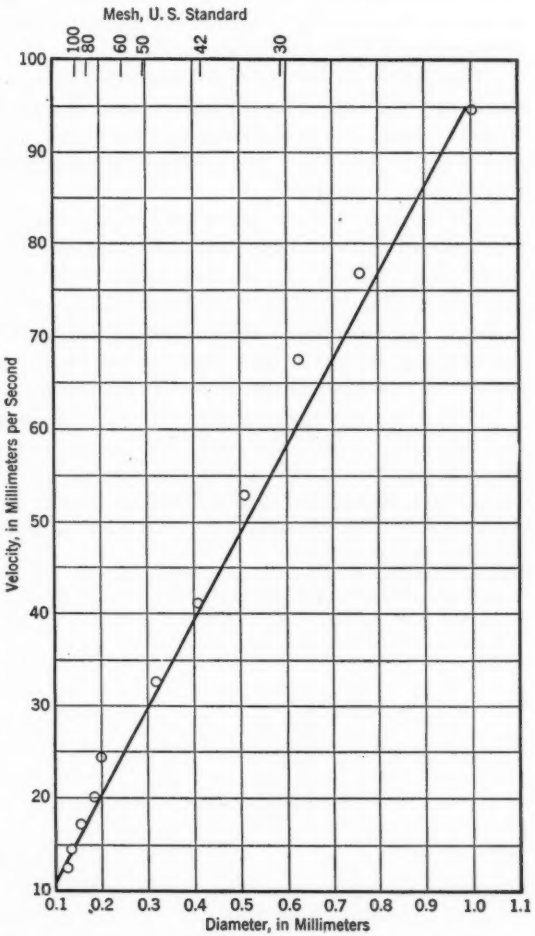

FIG. 20.—VELOCITY OF WATER IN WHICH SAND OF VARIOUS SIZES WILL SETTLE

The conventional circular radial-flow sedimentation tank has several serious faults. Not the least of these is the tendency toward unequal flow in the different quadrants. In some cases it is serious; and in relatively few cases has the problem been solved to the satisfaction of the operator. Wind velocities and wave action may be involved, but it is difficult to imagine any considerable uniform and constant effect from prevailing low wind velocities.

The results of an exhaustive series of observations made on full size units at the Southwest Sewage Treatment Plant, Chicago, Ill., have been reported

by Norval E. Anderson,[47] M. Am. Soc. C. E. A careful examination of the results reported would seem to indicate that deep tanks are unnecessary, and that there is plenty of room for improvement in the geometrical figure. It may be that, in order to conserve ground space and economize in construction, many tanks have been built deeper than necessary. Perhaps undue weight has been given to the theoretical period of detention while some of the more essential features have been overlooked.

The writer cannot refrain from entering a protest against the practice of drawing the sludge from the effluent end of a tank. Provision should be made to draw sludge, once settled out of the water, just as far away from the overflow weir as possible. This will provide for resedimentation, if necessary—a feature that is desirable.

The subject of trays is not quite new. Such provisions have not met with marked enthusiasm in the past. Further development in the details of design may bring more favor. In this connection, it might be timely to mention, once more, that handling a hundred million gallons of water or sewage per day presents more problems than merely building a number of smaller units. A multitude of factors, which may be overlooked as of negligible importance or easily met in a small plant, grow to major difficulties in large installations.

The author deserves a great deal of commendation for the logical and attractive manner in which the paper has been arranged; and for gathering, in one place, the pertinent elements on this subject. It serves to focus attention to the present status and recent developments in this extremely active and progressive field. It deserves amplification to several times its present length, and time for discussion.

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[47] *Sewage Works Journal*, Vol. XVII, No. 1, January, 1945, p. 50.

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DISCUSSIONS

FUTURE OF LAKE MEAD AND ELEPHANT BUTTE RESERVOIR

Discussion

BY ALBERT E. COLDWELL, WALTER B. LANGBEIN, C. S. HOWARD, CHARLES KIRBY FOX, H. V. PETERSON, L. C. CRAWFORD AND P. C. BENEDICT, CHARLES P. BERKEY, AND STAFFORD C. HAPP

ALBERT E. COLDWELL,[60] Assoc. M. Am. Soc. C. E.[60a]—This paper furnishes a valuable and long-awaited sequel to the author's paper on "The Silt Problem."[61]

To many readers it may have been somewhat of a surprise or mild shock to learn that the estimated life of Lake Mead is less than that of Elephant Butte Reservoir. These two reservoirs, however, have such similar lengths of life that it seems desirable to examine whether that most useful tool in sedimentation studies, the capacity-watershed ratio, may be applied effectively to indicate that such similarity is reasonable. These ratios are:

Reservoir	Acre-ft per sq mile
Elephant Butte	$\frac{2,634,800}{25,923} = 102$
Lake Mead	$\frac{28,632,000}{167,000} = 171$

For reservoirs in Texas and Oklahoma, Carl B. Brown,[62] Assoc. M. Am. Soc. C. E., has shown that, on the average, the annual storage loss for reservoirs with capacity-watershed ratios of 102 and 171 may be expected to be about 0.88% and 0.85%, respectively, of the original storage capacity. Applied to the two reservoirs in question, the resultant values of 114 years for Elephant Butte and 118 years for Lake Mead are very similar; and, after due considera-

NOTE.—This paper by J. C. Stevens was published in May, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1945, by D. C. Bondurant, John H. Bliss, Luna B. Leopold, Carl B. Brown, and G. E. P. Smith.

[60] Associate Engr. (Hydr.), Corps of Engrs., War Dept., Tulsa, Okla.

[60a] Received by the Secretary August 22, 1945.

[61] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 207.

[62] "The Control of Reservoir Silting," by Carl B. Brown, *Miscellaneous Publication No. 521*, U. S. D. A., p. 26.

tion is given to variation in location, climate, topography, and other conditions, these results appear to be in good agreement with the estimates of the author.

There is the possibility that either the capacity-watershed method or the linear method used by the author for estimating the lives of these reservoirs is too severe during the later portion of their lives. Large reservoirs in their early years have the ability to settle out virtually the entire inflowing sediment load, but with the passage of time the size of detention basin available becomes smaller and smaller until eventually a major portion of the inflowing sediment could be carried past the dam or deposited at elevations above the original pool level. McMillan Reservoir on the Pecos River seems to be a case in point. Much has been written, notably by the late R. F. Walter,[63] M. Am. Soc. C. E., about the effectiveness of the vegetative screen upstream from this lake, and since the writer has not had the opportunity to inspect closely these tamarisk thickets there is no desire on his part to minimize the value of this vegetation in preserving the capacity of McMillan Reservoir. The question is raised, however, as to what the condition of this reservoir would be in 1945 if the dense vegetation had not been present for the 30 years since 1915. This is an example of a small reservoir built on a large drainage area. Its capacity-watershed ratio has not at any time been more than 4 acre-ft per sq mile. Assuming an annual sediment inflow upstream from the vegetative screen of 600 tons per sq mile, approximately 183,000,000 tons of sediment (or, at 70 lb per cu ft, 120,000 acre-ft) had been deposited above lake level, in the lake, or had passed the dam in the 18-yr period from 1915 to 1932, inclusive. The capacity of the reservoir was depleted by only 5,000 acre-ft during this period. Why did not the portion of the remaining 115,000 acre-ft which passed by the vegetative screen become deposited permanently in the lake? Probably because the cross-sectional area of the remaining reservoir was so small that the velocities of sediment-laden inflows were not reduced any more by the lake than they had been by the vegetative screen. It has occurred to the writer that possibly if there were no vegetation above McMillan Reservoir its capacity would have now been additionally depleted only by an amount sufficient to insure that there would be sufficient velocity during high-water flows to carry all sizes of sediment past the dam. Floods on this river are often large enough to fill the remaining capacity of the reservoir in from one to two days, so it appears that, except for the storage of low flows, McMillan Reservoir virtually has become, so to speak, a run-of-the-river project, and possibly has been so for many years. It may be noted, as shown in Table 11,[64] that not during its life history has this lake had its capacity depleted by more than about 60%, even in its early years when the sediment problem appeared so severe. It would be interesting to know whether there are available profiles of the original and recent stream bed through this reservoir and upstream from it; and whether the volume of above-pool sediment deposits (which by now must be large) has ever been measured by valley cross sections or other methods.

[63] Discussion by R. F. Walter of "Silting of the Lake at Austin, Texas," by T. U. Taylor, *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1719.

[64] From the discussion by R. F. Walter of "Silting of the Lake at Austin, Texas," by T. U. Taylor, *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Table 8, p. 1722.

McMillan Reservoir has had its life history complicated by the construction of reservoir storage upstream, as eventually may be the case with all worthwhile reservoirs on heavily-laden sediment streams. Either as the result of acts of nature or man, it appears that the rate of sedimentation of reservoirs may not be a linear function, and that at some indeterminate time during their lives such a change in rate of sedimentation will act to their advantage. In

TABLE 11.—RELATION OF SEDIMENT DEPOSITS IN MCMILLAN RESERVOIR TO POTENTIAL ORIGINAL STORAGE CAPACITY AT VARIOUS ELEVATIONS

Date	Spillway crest stage (ft, on reservoir gage)	Capacity to spillway crest level, as if no sedimentation had occurred (acre-ft)	Sediment deposits below spillway crest level (acre-ft)	Ratio of sediment deposits to capacity (%)
1894	17.3	32,500	0	0
1904	17.3	32,500	16,000	49
1910	23.5	66,000	27,000	41
1915	27.0	90,000	48,500	54
1925	27.0	90,000	52,000	58
1932	27.0	90,000	53,500	60

the case of Elephant Butte Reservoir and Lake Mead this may occur too late to be useful in preserving any significant capacity for much additional time. However, at Lake Mead any storage thus accrued will be that much in addition to the 2.5 million acre-ft of residual storage which the author has shown will remain available as regulating storage. The study of sedimentation of reservoirs has gained such momentum, in the ten years since 1935 (and the problem certainly deserves such study), that it may now be time for investigators to evaluate the possibility that they are swinging with the pendulum toward the tendency for predictions of too pessimistic a nature. For instance, suppose the curve of Fig. 3 were extended with only slightly decreasing rates from the last two points on the curve for Elephant Butte. How much longer a life this reservoir could then be expected to have!

It is gratifying to note that the author has treated the reservoir for sediment storage as a necessary part of river-basin development rather than as a nuisance that should be built only if storage for other purposes fails because of difficulties with sediment. There comes to mind a remark by an engineer concerning a project to the effect that "They provided enough space to store the water, but forgot about a place to store the silt." At that time the writer was not particularly concerned with sediment problems, but the cogency of this statement has been realized since then.

This paper effectively calls attention to the existence of problem areas in large river basins. Similar areas have been found in the Arkansas River basin in Oklahoma as the result of investigations by the U. S. Engineer Department. Of the total water and suspended sediment flow in the Arkansas River as it crosses the Oklahoma-Arkansas state boundary line, the Canadian River furnishes approximately 68% of the sediment but only 22% of the water; and the Cimarron River furnishes approximately 11% of the sediment but only 5% of the water. These streams drain the arid and semi-arid area consisting of northeastern New Mexico, the Texas and Oklahoma panhandles, southwestern Kansas, and northwestern Oklahoma, with only a small portion

of their eastern watersheds being in subhumid territory. After entering the main stem of the Arkansas River the sediment-laden water from these rivers becomes diluted by the less turbid water of eastern tributaries, reducing the sediment concentration but not the total load. In the course of the study of duration of heavy sediment flows a relationship between tributaries and main streams was found, which is generalized in Table 12.

TABLE 12.—Relation Between Sediment-Carrying Characteristics of Tributaries and Main Streams

Relative flow rates	Headwater streams[a]	Main streams[b]
(*a*) Number of Days per Year Required for the Transportation of Each One Third of the Total Annual Suspended Sediment Load		
Highest flows	1	6
Intermediate flows	3	12
Lowest flows	361	347
(*b*) Approximate Range of Stream Stage During Which Each One Third of the Total Annual Suspended Sediment Load Is Transported		
Highest flows	Above bankfull	Above bankfull to half bankfull
Intermediate flows	Above bankfull to half bankfull	Half bankfull to quarter bankfull
Lowest flows	Half bankfull and below	Quarter bankfull and below

[a] Before the entrance of tributaries. [b] After the entrance of tributaries.

The significance of this table is that in the larger streams the transportation of significant amounts of sediment is a more continuous process than in the small streams. In conducting investigations of the possibility of venting, by density currents or otherwise, some portion of the inflowing sediment through flood-control and multi-purpose dams, it is necessary to watch closely the relationship between capacity of outlet works, size of permanent and temporary pools, and the incoming rate of flow even at times when such inflow is well below bankfull capacity of the stream.

It seems wise that the author did not extend the estimated lives of Elephant Butte Reservoir and Lake Mead on the basis of land management. When put in terms of average areal soil loss, the 198,000 acre-ft of annual deposit in Lake Mead amounts to an equivalent thin layer of less than one fortieth of an inch over the entire drainage area. Every particle of soil saved at the headwaters should be gratefully appreciated by operators of downstream projects, but the latter should plan, for reasons of safety, as if conservation measures would not decrease the sediment problem. The major premise for this belief is the difference in design criteria between small agricultural structures and major river projects. Because of economic considerations, terraces, small stock ponds, and other water-spreading devices seem seldom to be expected to care for storms of greater intensity than 25-yr frequency without failure. The damage done at the time of failure may cause the loss of much soil which had been saved during previous years. It is to be noted that, when a terrace near the top of a watershed breaks, each succeeding lower

terrace usually breaks. The concentration of water at the points of breaking tends to create a gulley. The larger reservoirs downstream, designed to withstand much more severe conditions, must be prepared to accommodate sediment inflows caused by these intense upstream conditions.

Any reservoir that has a life of from three to twenty generations of mankind should be considered as a national asset if it provides efficiently the benefits for which it was designed during its life. Surely there are few citizens who would say that the nation would now be better off if Boulder Dam had not been built, just because it might be full of sediment 135 years from now! Is there a possibility of extending the length of usefulness of reservoirs by turning sediment deposits from liabilities into assets? This appears to be a challenge to research engineers of the future. Soil that has been washed from Oklahoma farm land should be more valuable if left in Oklahoma than if deposited in the Louisiana delta, which already has all the rich soil it needs. After extensive sediment deposits have occurred in reservoirs, is there a possibility of controlling floods on them by upstream diversion or storage and then raising crops on this fertile land? Many of the valuable bottom-land farms of this nation are the result of sedimentation caused by flooding, and on them crops are grown of such utility that the danger of flooding is worth risking. L. C. Gottschalk[65] has shown that river silt acted as an effective fertilizer when mixed with loamy sand from a cultivated field in significantly increasing the yield of a crop of millet. This experiment was made under greenhouse conditions where there were no worries about drainage, volunteer vegetation, selective deposition of various qualities of soil in layers, and other problems which would be met in trying to cultivate the delta areas of a lake. Would these deltas be good sites for fertilizer plants? Could a water-tolerant crop, such as rice, be grown successfully? Possibly the answers to such questions as these will be available by the time the students of the future have the opportunity to assess the validity of present-day prophecies as to the future of reservoirs.

WALTER B. LANGBEIN,[66] Assoc. M. Am. Soc. C. E.[66a]—In projecting the short-term measurements of sediment transportation in order to estimate the life of major reservoirs, consideration of the "arroyo problem" in the Southwest is relevant. The question the writer wishes to introduce is: How representative of long-term conditions are the sediment loads now being measured in the major streams? The ultimate source of the sediment would seem to be the minor washes and their laterals which in the Southwest are only wet-weather courses. Muddy floods accompany and follow rain, but throughout the remainder of the year the stream beds are dry. Regarding their present condition, Kirk Bryan states:[67]

> "These ephemeral streams flow in valleys floored with alluvium from twenty to one hundred feet thick. The alluvium has been accumulated

[65] "Results of an Experiment on the Improvement of Sandy Soil with River Silt," by L. C. Gottschalk, *Special Report No. 3*, Sedimentation Sect., SCS, U. S. D. A., Washington, D. C., November, 1942.

[66] Hydr. Engr., U. S. Geological Survey, Washington, D. C.

[66a] Received by the Secretary September 10, 1945.

[67] "Erosion in the Valleys of the Southwest," by Kirk Bryan, *New Mexico Quarterly*, November, 1940, p. 228.

by the building up of the grade of the streams which, before the year 1880, ran in discontinuous shallow channels and during floods spread widely over the valley floors. After 1880 and continuing to the present, stream after stream has cut a deep channel, or arroyo, twenty to fifty feet deep into its valley floor or flood-plain. The mud excavated from these channels or arroyos is carried into the main rivers, and as the arroyos are widened by each successive flood, more mud is carried down. Because of the main streams, the Colorado, the Gila, and the Rio Grande, great reservoirs have been built, the increased load of mud will, within a few generations of men, fill the reservoirs and render them valueless for storage. This is the great 'arroyo problem.' How and at whose expense are these reservoirs to be protected?"

The significant and reasonably well authenticated fact is that these spectacular arroyos did not exist before 1870 or 1880. Many "old-timers" still remember the grassy bottom lands where fall hay cutting was a common practice and indistinct tree-lined channels meandered through valleys where today there is a vertical-walled gulley trenching the valley headward mile after mile. It is not uncommon at present to find well-defined roads leading to the walls of an arroyo and appearing on the opposite bank, showing that not many years have passed since white men forded the stream with teams and wagons or even motor vehicles. The Rio Puerco, now a dirty tributary of the Rio Grande, was once such a stream. Rio Viejo, as the Rio Puerco is fondly recalled by the natives, is well marked in many places by lines of dead cottonwoods. The entrenched Rio Puerco reaches headward for a distance of about 140 miles.

The present situation is plain, and there is sufficient evidence to discern the situation in Civil War days, but what induced the change?

A prominent theory advances the idea of a changing climate, suggesting that alternations from a more humid to a more arid climate in the Southwest have produced successive periods of sedimentation and erosion. Years of heavy precipitation, because of the increased growth of vegetation, are said to be times of minimum erosion by running water and consequently times of minimum silt transfer to the lower reaches of a stream. Existing conditions thereby are explained largely to have resulted from a change to a more arid climate. A widely accepted hypothesis, and certainly the most publicized, attributes the cause to man's abusive land practices—mainly overgrazing and the building of roads and trails. It is significant that both of these hypotheses attribute these abnormal gullies to the destruction of the vegetative cover, but differ as to the major cause of that destruction. Regional uplift causing rejuvenation of slopes is still another hypothesis that is offered to explain the change, although there is little to support and much to discredit such a theory.

It is a striking fact that the beginning of the arroyo cycle and the migration of cattle and sheep to the Southwest share a common date, and this circumstance has tended to encourage argument, one school arguing that the coincidence proves the guilt of man. Still another school argues that the cow is rather humble as a geologic agent and that, at worst, overgrazing was merely the trigger pull that timed the cutting rather than its cause. The engineer might well undertake as far as possible to retain a judicial view and establish any conclusions on documented evidence.

Physiographers point out that gullies are everywhere part of the landscape of arid regions and existed in the Southwest before 1870–1880. The gullies were more commonly discontinuous, as might be formed by local storms in eroding a small reach; the spreading of the water reduced the carrying capacity and the load was deposited no great distance beyond. Successive intensive storms served to carry sediments progressively downstream, but the movement was not continuous. There are indications that the present situation is quite different. Something has happened. The short discontinuous gullies in many basins have been integrated into a single gully system of spectacular dimensions.

Although fundamental, the matter has not been solved, mainly, the writer believes, because facts in the quantitative sense are few in number; and, indeed, the feeling is commonly expressed that conclusive evidence may never be established. A satisfactory working theory as to the cause would be valuable in framing a preventive and remedial policy and, more importantly, in avoiding measures that operate in opposition to natural adjustments.

Overgrazing can be indicted on its own account. The western range is a source of the major part of the nation's meat supply, but the use of the range must be adjusted to the growth of the forage if the yield is to be sustained. Thus the policy of government as expressed in the Taylor Act has been to discourage overgrazing through proper management of livestock in the public domain. The attitude here is not unlike that of a conservative landlord desirous of maintaining his equity.

Evaluation—through range surveys and mapping—of the grazing capacity of the several parts of the federal range, construction of stock tanks, drilling of wells, development of springs, and construction of fences to facilitate proper areal distribution of the grazing load in proportion to the capacity are all parts of modern range management. They can be justified in terms of meat production. The extent to which such management contributes to retardation of erosion cannot be estimated. Direct measures, such as the construction of check dams and similar structures intended to arrest the deepening or headward progression of gullies, have been widely attempted, with indifferent results in some places and an aggravation of the condition in other places. The situation has existed chiefly, it is believed, because (1) the introduction of such structures tends to upset the new balance that was being attained; and (2) the structures were built of local materials erosive by nature. Arresting the erosion of headwater gullies is attractive for its local benefits. In many places, notably Rio Puerco, streams have lowered their beds below the level of irrigable lands and subdrained the valley ground water. Any reduction of sedimentation in downstream reservoirs and elimination of other downstream damage effected are added, although highly significant, benefits. Nevertheless, no assured methods of arresting the erosion of deep gullies on widespread grazing lands of low unit value have yet been devised.

What bulk do the newly entrenched channels represent? No general surveys have been made to answer this question. Mr. Bryan and George M. Post in 1927[68] reported convincing evidence that the Rio Puerco had trenched

[68] "Erosion and Control of Silt on the Rio Puerco, New Mexico," by Kirk Bryan and George M. Post, Rept. to the Chf. Engr., Middle Rio Grande Conservancy Dist., October, 1927 (unpublished).

its valley for a distance of approximately 140 miles since about 1885. They estimated that about one third of the heavy annual sediment loads delivered to the Elephant Butte Reservoir are being derived from the extension and widening of this arroyo system. In addition, the trench now serves as a great flume to carry sediment directly to the main valley. From channel cross sections they estimated that between 1885 and 1927 about 395,000 acre-ft of sediment, or an average of 9,400 acre-ft per yr, was produced by the trenching of the Rio Puerco and its tributaries.

In an effort to gain further factual information on the valley trenches, the Geological Survey, in 1934–1936, mapped three gullies near Shiprock, N. Mex., in detail. Repeat surveys in 1944 revealed only minor enlargements, the greater part of the gullies in that region showing signs of quiescence rather than aggression. Lack of activity may suggest nothing more than that few, if any, cloudbursts visited the area during the elapsed ten years. Other surveys have since been made in several areas where repeat surveys are also planned.

Most of the gullies are approaching the heads of their respective valleys (if they have not already reached there), and before long must derive all their sediment loads from side cutting. When this stage is reached there is prospect that a decrease in annual sediment load can be expected, the simple explanation being that, as the arroyos widen, there is less tendency for the flood water to impinge on the banks and hence less opportunity for the stream to load up. The lower parts of many gullies already have reached the state where there is little side cutting and the banks are showing evidence of a reduced rate of cutting. Possibly the peak annual sediment load of these streams has already passed. Unless man intervenes, the end point can be reached only when the valley sediments are removed to the level of the entrenched stream.

What of the future? Should Mr. Stevens' extrapolation be straight lines, or convex upward or convex downward?

Field evidence, substantiated by artifacts, points to several periods of erosion and sedimentation. Mr. Bryan[69] places the period between 1100 A.D. and 1400 A.D. as another period of erosion, and the period between 1400 and 1870 as one of sedimentation. Evidently erosion, like rainfall, runoff, and other hydrologic factors, follows seemingly cyclic sequences. Field evidence cited suggests a slowing rate of removal of sediment in gullied valleys. However, there are many valleys as yet untrenched or where gullies are shallow and discontinuous. Aggression in these places may initiate a new and deeper erosion cycle, so the situation is not altogether clear.

However, to the extent that man has contributed to the cause or growth of valley trenching, the situation may in time be adjusted. The wider acceptance of controlled grazing should lead hopefully to some amelioration. To the extent that nature through climate is the cause, man's influence is problematical or impotent and forecasts are difficult. The writer believes that, to the extent that experience since the building of major reservoirs may have coincided with an aggressive phase of the erosion cycle, the long-term outlook may be more optimistic than other evidence suggests. Nevertheless, without

[69] "Erosion in the Valleys of the Southwest," by Kirk Bryan, *New Mexico Quarterly*, November, 1940, p. 229.

quantitative knowledge of the effect of modifying factors one hardly can find serious fault with Mr. Stevens' straight-line extrapolation.

C. S. HOWARD,[70] AFFILIATE AM. SOC. C. E.[70a]—As reported by Mr. Stevens in Part II of this paper, a large proportion of the sediment carried into Lake Mead comes from the sedimentary plateau that contributes a relatively small proportion of the water (64% of the sediment and about 20% of the water). Other computations based on the data given in Table 6 indicate that the sediment loads at the Bluff, Cisco, and Green River stations amount to about 54% of the load measured at the Grand Canyon gaging station; but the runoff for the three upper stations amounts to about 92% of the runoff at Grand Canyon. It is apparent that high concentrations and large quantities of sediment are brought into the Colorado River from unmeasured sources. The Little Colorado, San Rafael, Fremont, and Escalante rivers are the principal tributaries above Grand Canyon for which, unfortunately, sediment records are not available. It is desirable that records for those streams, and also for the Virgin and Muddy rivers, should be obtained.

Mr. Stevens has made estimates of the rate of sediment deposition in the various proposed reservoirs, using different weights per unit volume for the different reservoirs. It is doubtful if the available information is adequate for the accurate selection of such weights, but studies on the rate of sedimentation in Lake Mead should provide valuable information on the density of reservoir deposits. Such studies should include the determination of the volumes of deposits that have been formed in various parts of the reservoir and determinations of the sizes of the particles in those deposits. Correlation of the data on particle size of the deposited material and of the material carried in suspension will make it possible to predict future rates of sedimentation and, to some extent, the places of deposition.

In his concluding paragraph Mr. Stevens states: "The available data are pitifully meager, and must be greatly enlarged before any constructive program for relief can be developed." The data are meager and inadequate, not only as to quantity but also as to quality; but in the past few years steps have been taken to obtain more adequate data on sediment concentrations and loads. The reports[71] to which Mr. Stevens refers describe studies that have been made through a cooperative agreement between the Office of Indian Affairs, Bureau of Reclamation, Tennessee Valley Authority, Corps of Engineers, Geological Survey, Department of Agriculture, and Iowa Institute of Hydraulic Research. Through the efforts of this group, studies were made of the existing methods of sampling and analysis of the samples. Thereafter, improved types of samplers were developed for the collection of depth-integrated and time-integrated samples. With these improved samplers it is possible to obtain accurate information on the average sediment concentration; also the sediment concentration at different points in the cross section. In addition, a study of methods of

[70] Dist. Chemist, Geological Survey, U. S. Dept. of the Interior, Albuquerque, N. Mex.

[70a] Received by the Secretary September 20, 1945.

[71] "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," *Reports Nos. 1 to 9*, Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1940–1943.

analysis[72] of particle size led to the development of a bottom-withdrawal tube method that gives promise of being a useful procedure for determining the particle size of suspended and deposited sediment.

Data to supplement Table 4, for three years following 1941, are presented in Table 13. The totals and averages given are the results of combining both tables.

TABLE 13.—SUSPENDED SEDIMENTS IN THE COLORADO RIVER BASIN
(Millions of Tons)

Years ending Sept. 30	Location of Sampling Stations (See Fig. 5)						
	Green-river, Utah (1)	Cisco, Utah (2)	Lees Ferry, Ariz. (3)	Grand Canyon, Ariz. (4)	Bluff, Utah (5)	Col. 4 minus Col. 3 (6)	Virgin, Utah (7)
River	Green	Colorado	Colorado	Colorado	San Juan	Little Colorado	Virgin
Years of record	15	15	7	19	15	7	4
1942	31.0	33.8	....	229.6	69.1	....	....
1943	12.8	7.7	78.0	95.0	11.1	17	....
1944	23.2	16.1	77.3	97.8	17.8	21	....
Total *	355.5	274.3	1,085	3,626	631.7	331	12.4
Average *	(23.7)	(18.3)	(155)	(191)	(42.1)	(47)	(3.1)

* The total and average values include the years reported in Table 4.

Storage of water in a reservoir, as in Lake Mead, affects the chemical character of the water available for use below the reservoir. Studies of the quality of water above, below, and in Lake Mead show that the water released from the lake is more uniform in concentration than is the water flowing into the reservoir; the maximum concentration of the outflow from Lake Mead since storage began has been about one half the maximum concentration of the inflow. The average concentration of the outflow, however, has been somewhat higher than the average concentration of the inflow. Studies of the chemical character of the water in the inflow, outflow, and the lake have shown an increase in the concentration of certain constituents and a decrease in the concentration of other constituents. The increased concentrations are undoubtedly caused by the solution of the soluble constituents of deposits along the lake and the decreased concentrations represent chemical precipitation of certain constituents. The records indicate that, for the period from February 1, 1935, to September 30, 1944, 15,933,000 tons of solids were dissolved and 7,237,000 tons of solids were precipitated during storage in Lake Mead. It seems likely that the rate of solution will decrease as the soluble salts become less easily available, but the rate of chemical precipitation will remain constant and after a period of time there will be a small net decrease in the capacity of the lake because of these chemical processes. The volume of deposits formed by chemical precipitation will be a small percentage of the volume of deposits of suspended sediment from the inflow and it will be many years before the deposits from chemical precipitation will form an appreciable part of the deposits in the reservoir. There is, of

[72] "A Study of New Methods for Size Analysis of Suspended Sediment Samples," *Report No. 7* of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1940–1943.

course, stratification throughout the lake and the concentrations of dissolved solids in different parts of the lake vary from about 600 ppm to more than 900 ppm. Through regulation of releases of varying quantities of water from the upper and lower levels, it has been possible to maintain the average concentration of dissolved solids in the lake water at a nearly constant value.

CHARLES KIRBY FOX,[73] M. AM. SOC. C. E.[73a]—The countless silt-filled reservoirs extending from the Mediterranean area across southern Asia bear testimony that silt has been the bane of irrigationists since before the dawn of history; hence Mr. Stevens is to be congratulated for starting the discussion of this important subject.

The estimates of the silt carried by the Colorado River are many and varied, beginning at about 50,000 acre-ft of silt annually; but the later estimates have shown a decided tendency to "climb." Also, the weights of silt vary from 100 lb per cu ft (specific gravity 1.59; 2,178 tons per acre-ft) to 65 lb per cu ft (specific gravity 1.05; 1,415.7 tons per acre-ft) which would make a large difference in the estimated volume of silt carried (see Table 14).

TABLE 14.—COMPARISON OF ESTIMATES, SILT LOAD OF THE COLORADO RIVER

Item	Authority	Location	Period	Weight[a]	Annual Silt Load (Acre-Feet)		
					Suspended	Bed	Total
1	Dole and La Rue	Yuma, Ariz.	1895-1914	93	80,000	....	....
2	Mead, Schledcht, and Grunsky	Yuma, Ariz.	(Average)	100	90,000	12,000	102,000
3	Weymouth	Yuma, Ariz.	1909-1922	86	105,000	....	....
4	Davis	Yuma, Ariz.	(Average)	85	113,000	....	....
5	Davis	Boulder Dam	(Average)	85	88,000	....	....
6	Rothery	Yuma, Ariz.	1919-1921	86	94,800	....	....
7	Fortier and Blaney	Yuma, Ariz.	(Average)	....	119,000	19,000	138,000
8	Fortier and Blaney	Boulder Dam	(Average)	85	....	....	137,000
9	Grunsky	Yuma, Ariz.	1911-1927	100	94,500	....	....
10	Grunsky	Yuma, Ariz.	(Average)	100	....	25,000 to 30,000	....
11	Howard	Grand Canyon, Ariz.	1925-1928	86	152,700	....	....
12	Blaney and Howard	Yuma, Ariz.	....	....	94,000	....	....
13	Blaney and Howard	Topock, Ariz.	....	....	124,500	....	....
14	Blaney and Howard	Grand Canyon, Ariz.	....	....	154,500	....	....
15	Stevens	Lake Mead	....	65	....	....	198,000

[a] Dry silt, in pounds per cubic foot.

It is to be admitted, as Mr. Stevens states, that the lower basin of the Colorado is largely composed of soft easily eroded rocks, has many narrow steep canyons, high wind velocities, and is sadly overgrazed; but there is little apparent difference between the watersheds of the San Juan, Little Colorado, and Virgin rivers and the intervening plateau region or, for that matter, the Rio Grande, whose waters carry the highest percentage of silt and which, incidentally, is the only river whose silt load has been measured in a reservoir, except the great Grand Canyon, which some engineers call the world's greatest "silt mill."

[73] Cons. Engr., Los Angeles, Calif.

[73a] Received by the Secretary September 11, 1945.

A series of four diagrams relating to water and silt runoff are presented in Fig. 13. Supporting descriptive data are given in Table 15. In each case the lines in Fig. 13 represent the range of the records quoted. The yearly points are very scattered and in some cases, such as the Little Colorado and Virgin

TABLE 15.—WATER AND SILT RUNOFF; COLORADO

No.	River	Place	Area (Sq Miles)		Annual Average				
					Fig. 13(a)				Fig.
					Years		Water Runoff		Years
			Total	Alluvial	Range	No.	Acre-ft	Unit[a]	Range
1	Colorado.....	Yuma, Ariz.	244,800	....	1903–1939	37	14,100,000	57	....
2	Colorado.....	Grand Canyon, Ariz.	137,800	120,000	1923–1941	19	12,800,000	94	1925–1941
3	Colorado.....	Lees Ferry, Ariz.	107,900	95,000	1922–1941	20	13,300,000	123	1925–1941
4	Colorado.....	Cisco, Utah	24,100	18,800	1914–1941	22	5,800,000	238	1926–1941
5	Green.....	Greenriver, Utah	40,600	35,000	1895–1941	39	5,050,000	125	1926–1941
6	San Juan.....	Bluff, Utah	24,000	21,600	1916–1941	17	2,020,000	85	1927–1941
7	Little Colorado	....	22,100	18,800	1926–1941	16	242,000	11	1926–1941
8	Little Colorado	....	29,900	....	....	....	....	8	....
9	Virgin.....	....	934	....	1911–1941	23	161,000	160	1926–1941
10	Rio Grande...	San Marcial, N. Mex.	27,700	14,700	1897–1941	45	1,130,000	41	1915–1941

[a] Runoff in acre-feet per square mile. [b] Ratio of silt runoff; total acre-feet to acre-feet per [c] Percentage of silt to

rivers, so few points are available as to make it doubtful if the curves represent the conditions correctly. It will be noticed that the curves relating to the silt runoff in the Colorado River at Grand Canyon and the Rio Grande at San Marcial show a decided tendency to flatten during the periods of higher runoff. This is believed to be correct. In general, the water runoff during the silt-measured years is less than the water runoff during the other measured years.

The values in Table 4, Col. 6, are given as the difference between Col. 4 (Grand Canyon, drainage area 137,800 sq miles) and Col. 3 (Lees Ferry, drainage area 107,900 sq miles) indicating that the additional silt comes from the Little Colorado River. The difference in area is 29,900 sq miles. Mr. Stevens gives the drainage area of the Little Colorado River as 24,400 sq miles, whereas the Water Resources Branch of the U. S. Geological Survey gives it at 22,100 sq miles, a difference of 5,800 or 7,800 sq miles. Other data in the paper indicate that Mr. Stevens uses only the Little Colorado area in computing the silt, which varies from 7% to 14%, this range being outside the limits of the diagram in Fig. 13. It is immaterial which area is used as both stations are in the upper reaches of the Grand Canyon, with its steep grades, high water velocities in tremendous flood volumes, and steep canyon side slopes. It is very probable that the added silt is only that part which has been churned into suspension and that, for all practical purposes, the real bed silt is unmeasured.

It is doubtful if a reasonable estimate of the silt volume can be made from annual runoff data without reference to the ground, slope, and cover conditions

and, above all, to the flood periods. It is not believed that bed silt can be measured except in reservoirs.

The late A. H. Koebig once expressed the opinion to the writer that the average silt content of the Colorado River waters was between 8% and 12%

RIVER AND RIO GRANDE BASINS (SEE FIG. 13)

WATER RUNOFF (ACRE-FT)							SILT RUNOFF (ACRE-FT)					No.
13(b)			Fig. 13(d)				Fig. 13(c)					
	Silt Runoff		Years		Acre-Ft per Sq Mile		Years		Silt			
No.	Acre-ft	Unit[b]	Range	No.	Water runoff	[d]	Range	No.	Acre-ft	Unit[c]	%[c]	
....	....	....	....	....	....	....	....	....	....	....	....	1
16	12,500,000	92	1925–1941	16	12,600,000	92	1925–1941	16	142,000	1.04	1.12	2
16	12,300,000	111	1929–1933	5	11,300,000	104	1929–1933	5	130,000	1.21	1.17	3
16	5,600,000	223	1930–1941	12	4,930,000	206	1930–1941	12	13,000	0.35	0.24	4
16	4,010,000	100	1930–1941	12	3,450,000	87	1930–1941	12	16,900	0.38	0.42	5
15	2,020,000	85	1930–1941	12	1,880,000	78	1930–1941	12	31,200	1.28	1.65	6
16	242,000	11	1929–1933	5	292,000	13	1929–1933	5	41,500	1.87	14.2	7
....	....	8	....	....	....	10	....	....	....	1.38	10.50	8
16	143,000	153	1936–1941	4	153,000	164	1936–1941	4	2,200	2.35	1.16	9
27	1,108,000	39	1915–1940	26	1,105,000	38	1915–1940	26	13,300	0.52	1.27	10

square mile. [c] Ratio of water runoff to silt runoff. [d] Silt runoff in acre-feet per square mile. water in the runoff.

(by volume) and sometimes his measurements ran as high as 19% for hours and 41% for short periods. In view of the wide variations of the silt estimates and the practical impossibility of actually measuring the bed silt in a retarding basin, before the construction of the reservoir, the writer made a study of old maps, records of early explorers, logs of wells, subsidences, earthquakes, and other data which might throw light on the volume of the silt deposits and the length of time required for them to accumulate.

About 1936 some 128 maps of the head of the Gulf of California, dating from 1541, were obtained, traced, and carefully analyzed. Most of the maps were drawn to a small scale, but they were as good as other maps of a like period. The latitudes corresponding to Asia Minor (23°) and Spain (36°) were approximately correct but the intervening latitudes were often overstated by a degree or more on the old maps of all explorers.

Twenty-one maps follow the Friars' interpretation of the Indians' grunt they heard on the Iturbi expedition[74] in 1615 and show California as an island at the same time that Iturbi, Capt. Geronio Maquez, Franco Vasco, and others correctly reported that it extended to the vicinity of the present Indio. MacDougall[75] gives a map, by Sykes, of the northern part of the Salton Basin, clearly showing the old tilted shore line with elevations. Data relating to the southern area have been compiled from many sources; the shore line from the

[74] "Coleccion de Doucumentos de Inedious," by Joaquin Francisco Pacheco and Luis Torres de Mendoza, p. 30.

[75] "Salton Basin," by MacDougall, *Publication No. 193*, Carnegie Inst. of Tech., Pittsburgh, Pa.

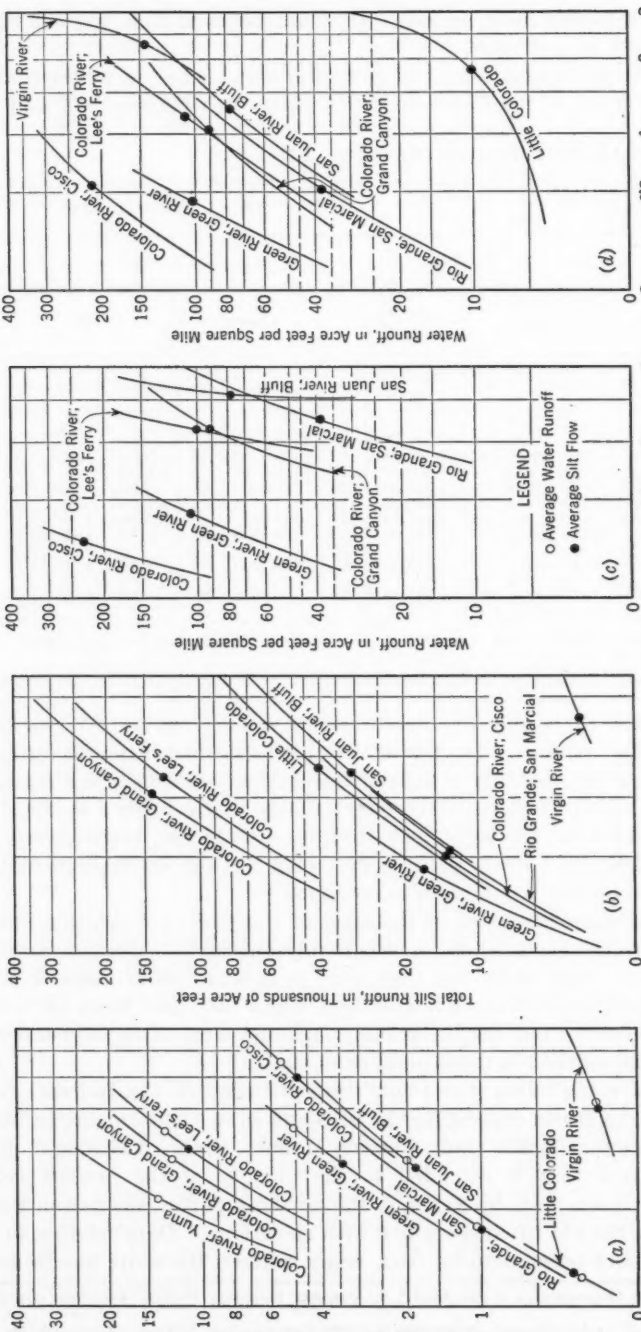

FIG. 13.—WATER AND SILT RUNOFF; COLORADO RIVER AND RIO GRANDE BASINS (SEE ALSO TABLE 14);

Castillo map of 1541 (Castillo spent thirty days surveying that area under instructions to show what he saw) clearly shows the mouths of the Gila River, Salton Basin, and the Laguna Salada.

Four of these maps and a mass of other data distinctly show the Gulf of California as extending to the vicinity of Indio, Calif. Logs of some twenty-five Colorado Delta Wells were located and plotted to obtain an idea of the depths of the deposits. Roughly they show, from the top downward: 125 ft of silt and 40 ft of gravel; 75 ft of silt and 30 ft of gravel; 40 ft of silt and 30 ft of gravel; and 180 ft of silt and 50 ft of gravel—an average of 105 ft of silt and 37.5 ft of gravel. It is certain that every time a great gravel stratum occurs its formation was preceded by a subsidence. The increased grade of the river enabled it to transport the heavier gravels, after which the silt was moved.

Since the Salton Basin was full of water when the Spaniards arrived it is quite probable that the previous subsidence occurred about one century before, or five centuries ago. Mexican and Peruvian correspondents have supplied the writer with Indian accounts of a very destructive earthquake occurring about this time. Also, in 1635, two earthquakes were recorded as being felt from Canada to Ecuador. It is quite probable that about this time the Salton Basin had lost enough water or weight to cause a readjustment which produced the tilted shore line.

As a check on the effects of the great changing weights some 5,000 to 6,000 earthquakes listed in the major catalogues were located and platted on a large map of the world. Then the centers of the great weight changing areas, with circles of radii—250, 500, 750, 1,000, and 1,250 miles—were platted. These circular arcs contained 21% of the world's land area, of which presumably one third is badly faulted and contained 93% of the plotted earthquakes. The changing volumes of some of the great lakes and the California debris (mining), with the earthquakes, were plotted with the same results.

From the aforementioned data, from a knowledge of approximate dates, from the depths of the silt and gravel deposits, and from an approximation of the areas, it is possible to approximate the average annual silt load of the Colorado River waters at something like a million acre-feet. Of course some silt has been carried to the Gulf and was not measured in the foregoing estimates. The estimate of 1,000,000 acre-ft may be a very rough approximation but is likely to prove more accurate than one which disregards or estimates the bed silt load which many geologists say amounts to from 100% to 400% of the suspended silt load, based on the measurements of many stream debris cones.

The Grand Canyon of the Colorado is about 250 miles long, narrow (the widest part being 12 miles), and deep (the deepest part being 1 mile), with a volume approximating 700 cu miles. The grade from Lees Ferry to the Bright Angel Trail, about 90 miles, averages 15.3 ft to the mile and the remainder of the canyon, 6.5 ft to the mile. These grades would cause very high velocities during floods, and the carrying power of water is supposed to vary with something like the sixth power of its velocity. Many geologists believe the canyon generally follows a series of faults. The Colorado River at Yuma usually digs 2 ft for each foot of rise in water levels.

The Elephant Butte Reservoir began to store water in 1915 and several silt surveys have been made of the reservoir site since then. Similar data should

be available for Lake Mead. The actual data most probably will show that the bed silt load has been greatly underestimated due to the effects of the silt accumulated in the Grand Canyon.

Reservoirs on any of the branches or the main river above Lees Ferry will tend to regulate the flow and, if large enough, will eliminate the great floods in the Grand Canyon which are undoubtedly the greatest source of silt, thus greatly prolonging the useful life of Lake Mead.

H. V. PETERSON,[76] ASSOC. M. AM. SOC. C. E.[76a]—In furnishing an estimate of the probable useful life of these two reservoirs, the author has forcefully called attention to the insecure future of numerous western irrigation projects caused by the continuous reduction of reservoir storage capacity through silt deposition. Although the public in general and engineers in particular may claim just pride in the accomplishment of creating such magnificent structures as the two dams in question plus many others, an even greater task remains if the maximum possible benefits from such construction are to be attained.

Doubtless there will be disagreement with the author's estimates. An optimist might enumerate factors that would tend to show greater reservoir lives than the estimates given and, just as consistently, one of opposite attitude might find reasons for believing them to be less. That the reservoirs will be filled at some future period is patent to all and, whether the author's estimates are correct to within a year, a decade, or even a century, it is obvious that the usefulness of these great structures is being seriously imparied at an alarmingly rapid rate. What will perhaps startle many readers is the author's statement concerning the rather insignificant effect that man's efforts in land management can produce in prolonging the life of the reservoirs. It is with this phase of the subject that this discussion will deal.

During the past few years much thought has been given to the problem of erosion, and many discussions on the subject have appeared in both the popular and technical press. Doubtless, there is some reason for pride in the accomplishment during this period; but that only a beginner's step has been taken in solving the problem, particularly as it applies to the arid western region, is evident from the author's statement (see heading, "Prolonging the Life of Lake Mead"): "It is doubtful if * * * sediment being carried annually into Lake Mead could be reduced as much as 10% by all the proved practicable methods of land management known today even though lavishly financed." Since silt movement in the Colorado River is typical of much of the arid West, it is implied that similar management plans applied to other streams in regions would have as little effect. The statement casts a dark shadow over the future of western development and irrigation, and certainly leaves uncertain the value of the heritage bequeathed to coming generations; but, in the writer's opinion (much as he dislikes admitting it), there is little evidence at present to disprove or even temper the author's conclusion.

Is this an indictment on engineers and conservators for their failure to develop a practicable scheme for controlling erosion, at least such portions of it as are amenable to man-made controls, or does it mean (as the author implies

[76] Geologist, U. S. Geological Survey, Los Angeles, Calif.

[76a] Received by the Secretary October 2, 1945.

elsewhere) that, in endeavoring to establish such controls, natural laws will be encountered because the forces of natural erosion so outweigh those attributable to man's activity that the best that man can do in the way of land management can have but minor effect on silt movement? An answer to the question is important. A possible extension of only 10% in the life of the reservoirs furnishes little incentive for investigation or for setting up any program looking toward future reduction of silt movement by the streams.

An examination of the alluvial fill found in the tributary valleys of the Colorado River and the Rio Grande (and, in fact, practically any of the valleys of the arid West) reveals that the basins have passed through alternate cycles of alluviation and degradation at various periods in their past history. Cycles of alluviation result where the products of weathering—including silt, sand, gravels, etc.—are delivered to the valley floor at rates faster than the transporting agencies—including wind and streams, but mainly streams—can remove them, and conversely degradation cycles occur where opposite conditions prevail. Evidence indicates that, although weathering under desert conditions is relatively slow, it is nevertheless constant; and therefore it may be assumed that the cycles resulted from changes affecting the transporting agencies.

The effectiveness of streams would be enhanced by increasing the stream gradient or by increasing the total volumn of flow, either through higher precipitation over the drainage basin or through decreasing infiltration (which increases the percentage of immediate runoff). Because streams are by far the most important transporting agency, any such conditions would be favorable for starting a degradation or erosion cycle.

According to numerous documents, reports, histories, and testimony of many persons still living, the West has passed from a cycle of alluviation to one of degradation since about 1870; or since white man's migration to this region. The resulting change in the landscape has been startling. Available evidence shows that, prior to white man's occupancy, most western valleys presented a smooth unbroken floor in which the stream at ordinary stages followed a meandering, shallow tree-lined or brush-lined channel. Flood flows that exceeded the capacity of the channel spread slowly over the valley depositing silt and doubtless loosing an appreciable percentage of its volume to ground-water percolation. The reports that many of the present barren valleys once carried permanent streams and supported large tracts of luxurious grass that yearly was cut as hay are not inconsistant with this picture, although the contrast with present conditions makes it almost unbelievable.

Today most of these same valleys are scarred by ugly gullies that cut through almost their entire length. Ground water, which once fed the permanent streams and nurtured the grassy flats, has been drained, leaving seared strips of barren waste adjacent to either bank of the gully. Flood flows now concentrate in the channels and instead of loosing silt, as they once did, they take on additional loads from the caving banks with the result that at times the discharge might appropriately be called watery mud rather than muddy water. It is reliably reported that 111 of the 115 major tributaries of the Colorado and Green rivers above Lees Ferry are in this condition. Below Lees Ferry the Virgin River, Kanab Creek, Johnson Wash, and practically every smaller tributary valley is likewise trenched. Tributary valleys of the Rio Grande are in

similar condition, the valley of the Rio Puerco being traversed by one of the best known and most spectacular of all the gullies. It is the silt derived from these valleys that is destroying the two reservoirs under discussion and unless some means can be found to prevent the continued movement of this silt any future reservoir must also be destroyed.

It is not the purpose of this discussion to take sides in the arguments concerning the cause of this recent gullying. From the observation and experience gained through a lifetime residence in the West (the past several years of which have been spent in detailed investigations having to do with land and water utilization problems in many of the arid tributary valleys of both the Colorado River and the Rio Grande), the writer has concluded that no single cause can be held responsible for the abrupt change from one cycle to another. Land misuse, principally overgrazing, climatic changes resulting in an increase or a decrease in precipitation, and tectonic movements which bring about steepening of the stream gradient have each been advanced as adequate to institute the change; but the evidence supporting any of them is too indistinct and at times too contradictory to be conclusive. It would appear that a combination of all these processes, with possibly others not yet recognized, might more consistently be considered the basis for the change; and the engineer, in his attempts to prolong the life of the reservoirs, should recognize this fact and plan his treatment accordingly. Under these conditions, what avenues can be pursued and what results can be expected from his efforts?

Controlling silt movement by correcting land misuses is especially appealing to those who subscribe in whole to the theory that abuses instituted by man are entirely responsible for the destructive erosion now taking place. Overgrazing is cited as the most flagrant of these abuses and any one who has inspected large areas in both the Rio Grande and Colorado basins, as the writer has, must admit the seriousness of this accusation. What steps will be necessary to correct the condition and how soon and to what extent tangible results can be expected are questions of vital importance in considering the merits of a corrective program of this nature.

Extremists along these lines have advocated elimination of all grazing, contending that the value to livestock of the forage produced is far less than the protective value of the vegetative growth in stabilizing and holding the soil mantle in place. Such a stand, however, overlooks the political repercussions sure to result from such a violent disruption of the society and economy centered about the livestock industry within these basins just as it ignores many other practical aspects of the problem. As an example of what such a program would involve, the case of the Navajo Indian Reservation may be cited. This reservation is considered to be one of the worst silt producing tracts within the Colorado basin. Almost its entire area is underlain by easily eroded, friable, poorly-cemented sandstones and shales. Scores of washes and arroyos, including such well-known silt carriers as Moenkopi Wash, Polacca Wash, Oraibi Wash, Chinle Creek, Navajo Creek, and Chaco River, drain these unstable areas. Each is deeply gullied and together they contribute thousands of tons of silt annually to either the Colorado, the Little Colorado, or the San Juan rivers. Some 60,000 Indians live within the Reservation, the great majority of whom are dependent on the livestock industry for their living.

Overgrazing in many parts of the reservation is apparent and doubtless is contributing in considerable measure to silt movement. To cure this ill by eliminating grazing would mean moving and rehabilitating this entire group—a task approaching in cost and probably exceeding in complexity the construction of another dam on the Colorado River. Such a step is obviously impracticable under present circumstances. Even a reduction in livestock numbers sufficient to correct the most aggravated areas of overgrazing must be accompanied by some provision for maintaining the present standards of living of the Indians; otherwise severe opposition can be expected not only from the Indians but from the public at large.

The problems of other graziers within the two basins must also be appreciated if a program of range improvement is to be successful. In general, livestock operators of the West are individualists of the most independent type. Through skill, daring, hard work, and dogged perseverance they have learned to wrest a living from these forbidding desert reaches, and it is the writer's impression, gained through a rather wide acquaintance among them, that they have no intention of allowing any one to interfere with their chosen method of obtaining a livelihood. They consider their prior interest in any program having to do with river developments equivalent to or superior to the interests of the irrigator, and any plan that seeks arbitrarily to reduce or eliminate grazing within the basins is doomed to failure unless this attitude is fully appreciated and adjustments made for it.

It should not be inferred from the foregoing that stockmen have taken a narrow attitude in regard to range management. The stand taken by most independent operators and owners, and by public officials engaged in administering grazing on the public domain, well agrees with the author when he states (see heading, "Prolonging the Life of Lake Mead: Land Management"):

> "If grazing areas are managed to produce their maximum forage yield the erosion prevention feature will have been provided for automatically; that is, the prime objective in management of range land should be to furnish as much food as practicable for livestock on a sustained yield basis."

These men are fully conscious of the over-all aspects of range utilization. They appreciate that an overgrazed scar is a potential source of debris, capable of destroying reservoirs as well as destroying forage; but they are not of the type who will "go overboard" to cure that scar by retiring from the livestock business. They will cooperate, and they are cooperating, to bring about better range conditions so far as practicable; but they refuse to be concerned over erosion cycles, or to assume responsibility in inaugurating them.

Criteria are lacking for judging the results that may be expected under best range management as it relates to silt movement. Optimists are inclined to point out certain ungrazed areas and experimental plots showing a minimum of erosion as good examples of what will obtain under perfect range management. However, range experience indicates that a long period of careful husbandry will be required before any appreciable reduction in silt movement from a wide area directly ascribable to this cause can be noted.

During the past few years the writer has had the opportunity to inspect a number of fenced plots in which grazing has been excluded for periods ranging

up to 10 years. These areas indicate that, while improvement in vegetative cover may be noticeable within a relatively short period, the actual establishment of a growth sufficient to form a protective stabilizing soil cover will be a slow tedious process requiring years, perhaps decades, for its fulfilment. Actually vegetative growth within these desert areas is so dependent on available moisture that other factors are of distinct minor significance. For example, if a person unacquainted with desert vegetation had examined some of the major grazing areas in both the Colorado and Rio Grande basins during the extremely wet winter season of 1940-1941, he would have scoffed at the mention of overgrazing or of inadequate cover protection; but had he returned a few months later the grim effects of drought would have been fully apparent in the sparsity of grass and forage and in the large percentage of the land exposed to the erosive action of wind and rain. Any one familiar with the desert is fully aware that drought is a greater enemy of plant growth than overgrazing. Perhaps by careful husbandry a cover capable of withstanding these periodic and prolonged dry spells can be established over a substantial part of both river basins, but the proof of this remains to the future. To date the prospects look discouraging.

Even assuming that through careful husbandry and management the range will eventually be maintained in the best possible condition consistent with judicious grazing use, the question as to what effect this will have on silt reaching the main rivers still remains unanswered. Unfortunately, no data are available on which to base even an estimate of the effect. So far as the writer knows there is at present no demonstration or experimental plot of any appreciable size within a typical desert portion of either basin where an effort is being made to study and gather data on this phase of the erosion problem. A few preliminary and fragmentary records have been obtained from one or two experiment plots established for this purpose but unfortunately the stations were discontinued before any conclusive evidence could be gathered. Without such information the conservator and the engineer must work under a tremendous handicap.

In theory a combination of structural treatment and range management would appear to offer the most effective means for preventing erosion and silt movement on the valley floors. Structures of the proper type could be used temporarily to store the silt and form it into a suitable seed bed. Thereafter the plant cover would hold it in place and by proper manipulation this cover would continue to trap additional silt until the entire gully system was refilled and healed. Experience, however, does not confirm this theory; at least it has not in a number of rather elaborately treated eroded areas located on the floors of tributary valleys of both the Colorado River and Rio Grande, which have been examined by the writer during the past few years. Except where there has been a continuous effort to maintain, correct, and improve these structural installations, and the exceptions are few, the result obtained in the way of soil stabilization and increased vegetative growth has been highly disappointing. In some instances actual damage and increased erosion have resulted from the structural treatment. Poor construction, poor design, and in some cases poor planning must share the responsibility for the failure of such treatment, along

with lack of maintenance and an improper appreciation of the forces involved in the erosion processes.

A successful program for controlling silt, holding it in the valleys and out of the reservoirs, will be found, but it obviously must be based on bolder, more imaginative lines than have heretofore been thought of. Myriads of problems in engineering, geology, botany, agriculture, and the arts of land and livestock management are involved, and available information with which to solve them is woefully meager. Information is needed on the type and design of structures that can be depended on for consistent performance under rigorous desert conditions. More should be known of what can be expected from a coordinated program of structural treatment and strict range management. Methods of range control and rehabilitation need to be studied and perfected. Regions and rock formations producing excessive amounts of silt need to be segregated and analyzed as prospects for special treatment. It has been suggested that erosional experiment stations should be established where treatment programs could be developed and tested on a small scale. The similarity of the erosion problem in so many of these valleys makes this proposal both consistent and desirable. In short, what is needed, as the author stresses, is more information on the entire erosion problem. With investment on the rivers, installed and in prospect, running into hundreds of millions of dollars, it would seem consistent to spend a small part of the development cost in measures aimed at protecting the investment by prolonging its life.

L. C. CRAWFORD,[77] M. AM. SOC. C. E., AND P. C. BENEDICT,[78] ASSOC. M. AM. SOC. C. E.[78a]—This comprehensive analysis of available sedimentation data in the Colorado River basin is timely because of the increasing recognition of the need for such studies and data throughout the United States. The general considerations and conclusions have attracted considerable interest, and the indicated paucity of data is of widespread concern not only in the Colorado River basin but elsewhere. In view of the measurements of suspended sediment loads and related field work done in Iowa, with improved sampling equipment, some information concerning this program may be of interest or value in connection with the discussion of this noteworthy paper.

Mr. Stevens refers to certain reports that have been prepared and published as a part of a project involving "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams."[79] This project was inaugurated in 1939 under the sponsorship of an Interdepartmental Committee composed of representatives of the Tennessee Valley Authority, Corps of Engineers, Department of Agriculture, Geological Survey, Bureau of Reclamation, and Office of Indian Affairs in cooperation with the Iowa Institute of Hydraulic Research. Experimental models of depth-integrating and point-integrating sediment samplers, designated US D-43 and P-43 sediment samplers, respectively, were completed in 1943. Subsequently, both of the experimental models and several duplicates of the D-43 sampler have been

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[78] Hydr. Engr., U. S. Geological Survey, Iowa City, Iowa.

[78a] Received by the Secretary October 4, 1945.

[79] "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," *Reports Nos. 1 to 5, 7, 9*, Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1940–1943.

subjected to tests under practical field conditions, alongside of, and in comparison with, other sediment samplers in current use, to determine the practicability of the new samplers, and to evaluate their sampling accuracy with respect to other samplers.[80] The comparative test data submitted in reports by thirteen field offices of various agencies have been analyzed, and a progress report[81] reproduced by the St. Paul Office of the U. S. Engineer Department. The development of a satisfactory bed-load sampler has not, as yet, been undertaken by the Committee.

The results of the interdepartmental investigation, for which the Corps of Engineers and the Geological Survey have been providing the active leadership in recent years, lead to the conviction that the needed extension and improvement of sediment data indicated by Mr. Stevens can be materially aided by the adoption of standard equipment, and methods of measurement somewhat comparable to the commonly accepted procedures[82] of stream gaging. Therefore, and as a part of the program to standardize sampling equipment among the interested agencies participating in the joint investigation at the Hydraulics Laboratory in Iowa City, the D-43 sampler was selected for use in connection with the determination of daily suspended sediment loads in cooperation with the Iowa Geological Survey and the Rock Island Office of the U. S. Engineer Department at four stations on the Iowa, Cedar, and Des Moines rivers in Iowa. This field work afforded an opportunity to obtain further information on the field operation of the samplers. A summary of the preliminary reports of sediment discharge (in tons) for the water year 1943–1944 (ending September 30, 1944), at the Cedar Rapids, Iowa, and the Iowa City, Iowa, gaging stations, with drainage areas of 6,640 and 3,230 sq miles, respectively, is as follows:

Month	Cedar River at Cedar Rapids (tons)	Iowa River at Iowa City (tons)
1943—		
October	3,010	3,270
November	2,000	3,670
December	1,760	1,320
1944—		
January	2,950	39,700
February	26,900	115,400
March	104,500	282,100
April	61,100	328,500
May	262,400	1,121,300
June	441,200	561,800
July	68,000	131,700
August	17,600	26,100
September	8,410	4,790
Total	999,330	2,619,650

[80] "Preliminary Field Tests of the US Sediment-Sampling Equipment in the Colorado River Basin," by P. C. Benedict, mimeographed rept., U. S. Geological Survey, Iowa City, Iowa, April, 1944.

[81] "Comparative Field Tests on Suspended-Sediment Samplers," by M. E. Nelson and P. C. Benedict, mimeographed rept., U. S. Engr. Sub-Office, Hydr. Laboratory, Iowa City, Iowa, December, 1944.

[82] "Stream-Gaging Procedure," *Water-Supply Paper No. 888*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1943.

These quantitative records of suspended sediment loads are based on continuous and systematic field measurements made throughout the year, using the equipment and methods indicated in the reports prepared under the direction of the Interdepartmental Committee. The results show that the yearly suspended sediment load carried in the Cedar River at Cedar Rapids amounted to slightly less than 1,000,000 tons or 150 tons for each square mile of drainage area above Cedar Rapids; whereas, more than 2,600,000 tons, or 810 tons per sq mile for the year 1944, were determined at the gaging station on the Iowa River at Iowa City.

Studies made at the Hydraulics Laboratory of all available information on the density of deposited sediments (which included a review of the data for the Colorado River and the Rio Grande basins) indicate that the average weight of 1 cu ft of deposited sediment from the Iowa River may be taken as 60 lb.[83] On this basis, the suspended sediment measured at Iowa City during the year that included the flood waters of May and June, 1944, would occupy an equivalent space of about 2,000 acre-ft.

It is interesting to note that, although the characteristics of the Iowa River basin are widely different from those found for rivers in the southwestern United States, an average suspended sediment discharge of 810 tons per sq mile per year was measured during a flood year in comparison with the average sediment inflow (suspended and bed load) of 1,600 and 1,350 tons per sq mile per year obtained by Mr. Stevens in his analysis of the average total load entering Elephant Butte Reservoir and Lake Mead, respectively.

During the floods that occurred in Iowa in 1944 and 1945, sediment samples were also collected with the 50-lb D-43 sampler at three to six stations in the water cross section at a number of regular gaging stations. The samples were taken, in general, at or near the crest after the completion of a current-meter discharge measurement. It is recognized, of course, that data thus provided do not adequately define the flood or sediment transportation potentialities of these or other streams in similar drainage basins. However, this type of field investigation does provide a convenient and inexpensive means of reconnaissance in connection with the location of desirable sediment sampling stations. For example, a sediment discharge rate of 2,030 tons per day per sq mile has been measured for a drainage area of 130 sq miles. The maximum average concentration of sediment by weight in a cross section was 23,600 ppm (2.4%) for a drainage area of 2,800 sq miles.

Such quantitative facts together with the daily suspended loads at regular stations, if obtained over a period of years, will be increasingly helpful with reference to the author's comment concerning the pitifully meager status of available sedimentation data for the proper consideration of a variety of problems relating to agriculture, conservation, land-use practices, and to the future of reservoirs. Furthermore, standardization of equipment and methods will assist in consolidating engineering effort (the Hydraulics Division of the Society has a joint committee on sedimentation in reservoirs) on all phases of

[83] "Density of Sediments Deposited in Reservoirs," by E. W. Lane and Victor A. Koelzer, *Report No. 9* of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," U. S. Engr. Sub-Office, Hydr. Laboratory, Iowa City, Iowa, 1943, p. 38.

sedimentation data, the need for which has been effectively demonstrated by Mr. Stevens.

CHARLES P. BERKEY,[84] HON. M. AM. SOC. C. E.[84a]—Although the principles involved in the paper by Mr. Stevens are well known, it is not certain that the implications are fully appreciated by many even in responsible relation to them. The factual data had been long known to the writer, but no statement before this one had brought so forcibly to mind their importance and bearing on long-range planning. This does not apply only to Elephant Butte or Boulder, but to the entire enormous undertaking of recovering lands by irrigation. The United States has virtually set up an empire on impounded and redistributed water. The nation is encouraging development, on a scale never before attempted, of lands that are almost worthless except for the waters that can be delivered to them by the works of man. There is building up, through settlement and new population, a line of industries foreign to the normal resources of the region and they appear to be doomed to a limited life.

The situation reminds one of the "finds" of archeologists indicating that civilizations have flourished while climatic conditions were favorable and have disappeared when the skies withheld rain. The factual accounts of R. Pumpelly [85] and E. Huntington [86, 87] in western Asia abundantly prove that even civilized peoples of some ingenuity could not withstand persistent drought. Eventually, of course, a cure was found and heralded as a new kind of salvation. Men had learned to impound such waters as there were and to carry them to their parched lands. On one occasion the writer found evidence of settlements along the streams which, in an earlier epoch, had been completely obliterated by steadily increasing aridity. Even fishing and hunting, which must have been the chief occupations of these people, had dwindled to the vanishing point while the winds drove drifting sands over their dwelling places until the last man gave up. Still later, in another age and in another valley nearer to the mountains, where mountain streams still carried waters far out into the adjacent desert, they learned to irrigate a little land; but the people of that valley also had to give up when their ditches ran dry. One could trace the faint lines of the distribution system to their garden plots where not a soul remained in this day to till them. In yet another valley its people built a dam to impound the dwindling water supply; but even that failed to save them and the surrounding structures built of granite lie around in ruins. The people who built them are gone.

These instances, to be sure, are not quite parallel to the present effort to reclaim lands; but in capturing silt-laden rivers engineers are facing an equally devastating natural danger. So much silt is impounded with the water that it literally crowds out its assumed long-time promise. This danger is much more imminent than climatic change.

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[84a] Received by the Secretary October 15, 1945.

[85] "Explorations in Turkestan," by Raphael Pumpelly, Washington Carnegie Inst. of Washington, 1903–1905.

[86] "The Pulse of Asia," by Ellsworth Huntington, Houghton Mifflin Co., 1907 and 1919.

[87] "Civilization and Climate," by Ellsworth Huntington, Yale Univ. Press, 1915 and 1924.

Effort to use water on desert lands is not a new adventure by any means; but a program involving development of a great region—inviting thereby a large new population under conditions that carry elements of certain future destructive encroachment in limited and computable time—that is new. Not only is it new, but in some of the implications it is fairly astonishing. As a public policy probably nothing like it has ever been faced—certainly not on such a scale. The nearest thing in that respect was the settlement of the western high plains in earlier days by people who believed that these dust-bowl lands could be farmed in the same manner as those they came from in the Central Mississippi Valley, and no voice was raised to warn them. That was to be a vast and prosperous empire, too.

For the first time, after reading this paper, the long-range significance of the suffocating effect produced by accumulating silt in all these reservoirs was borne down on the writer. He had been so much taken with the fine things being done that he had not fully appreciated the fact that the program carries elements of destruction sure to bring some kind of ending. It was always evident, of course, that there were severe limitations, but it was too easy to overlook or belittle this element of damage from within.

The experience of founding, in difficult surroundings, settlements which finally grew into influence and power is not new; and neither is their decline, and even their ending. In the past, however, none of them carried along, with the agents that built them up, such relentless elements of destruction as in the present reclamation of arid lands. The astonishing thing is that the life of these relief works promises to be so short. One could forget it if the time vista were indefinite, or if there were promise of a thousand years. In that time most human subsistence and economic lines take new turns and become adjusted; but in some of these projects, typical of the average more or less, the beginnings of decline loom already and will certainly grow into a serious problem in three or four generations. One wonders how many settlers gathering around these projects appreciate what it means.

Of course, if one is able to divorce his interest from the future, there is nothing to worry about. In this generation, and the next and the next, an upgrade can be maintained. One can claim (and it is true) that much has been added to the world; but the longer range view in this field, as in many others, is threatened by apparently incurable ailments and this one of slowly choking to death with silt is the most stubborn of all. There are no permanent cures. All are familiar with the simple elements of the matter; but the writer had faced all of that evidence without feeling greatly disturbed. The thing that did most to create a real shock was the figure on Bridge Canyon. The only time that site was visited, after mushing through thirty miles of silt-laden soupy river to get there, such close attention had to be given to the dam site itself that the silting of the proposed new reservoir was almost forgotten; but the paper by Mr. Stevens brings it out, and is a fair and timely reminder, that this is a problem of the first magnitude. Although there are ways of postponing the end stages of these developments, there is no permanent cure, for the final contest transcends engineering skill. It has all the elements of a last stand.

STAFFORD C. HAPP,[88] ESQ.[88a]—The approximate sediment loads of the Rio Grande and Colorado rivers are fairly well known from existing records; and there is no apparent reason to doubt the order of magnitude of Mr. Stevens' computations of past and probable near-future sedimentation rates. Projection of these rates so far into the future, however, seems questionable unless greater allowance is made for the normal effect of delta growth in reducing the rates of capacity loss. There is also need for clarification of the values given for rates of sediment delivery from major Rio Grande tributaries above Elephant Butte Reservoir.

Delta growth is one of the most important factors in reservoir sedimentation. The history of Lake McMillan is perhaps the best known example. Growth of salt cedar has often been cited as the cause of the remarkable decrease in the sedimentation rate in Lake McMillan, but this was secondary to the effect of delta development. The salt cedar has been very effective as a sediment screen on the top-set delta beds, because there the capacity of the Pecos River channel has been reduced by sedimentation and most of the flood waters are forced overbank through the salt cedar thickets. The rate of capacity loss would have been reduced by the delta development alone, although there is no doubt the salt cedar has been an added influence of considerable importance. Elephant Butte Reservoir had a much larger capacity relative to drainage area and inflow, so that its delta has been deposited more largely within the original reservoir basin, and the rate of capacity loss has been retarded less rapidly; but it is only a matter of time until the Elephant Butte delta will reach such size that its further growth will be largely outside the reservoir proper. Salt cedar was growing abundantly on the Elephant Butte delta in 1936 and 1941, although less conspicuously than at Lake McMillan because interspersed with larger proportions of willows and cottonwoods, and there undoubtedly will be sufficient vegetation to stabilize most of the Elephant Butte delta deposits.

Surveys in 1941 showed that the effect of delta growth had extended approximately 15 miles upstream from the head of Elephant Butte Reservoir, to the lower end of the Middle Rio Grande Conservancy levees. From 1936 to 1941, the average annual deposition in this 15 miles of valley was about 12.5 million tons. Less complete, but apparently representative, survey data indicate that the 1936–1941 rate of sedimentation in the lower half of this area was about $2\frac{1}{2}$ times as great as that from 1914 to 1936. The rate of increase in sedimentation was probably even greater in the upper half of the area, but data are not adequate for quantitative comparisons. The average rates of sedimentation in the reservoir, according to Mr. Stevens' paper, were about 27 million tons annually from 1914 to 1935, and about 13.5 million tons annually from 1935 to 1940. Suspended loads in the Rio Grande at San Marcial, approximately at the head of the reservoir, were about 11 million tons annually for both the 1935–1940 and 1936–1941 periods; so it is reasonable to compare the 1914–1935 and 1935–1940 rates of reservoir sedimentation with the 1914–1936 and 1936–1941 rates of sedimentation immediately above the reservoir.

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[88a] Received by the Secretary October 22, 1945.

Such comparisons show that sedimentation in the area of delta influence, upstream from the reservoir, was about 48% of the combined total from 1936 to 1941, but only about 15% from 1914 to 1935.

The rate of sedimentation in the delta area was greatly increased by several channel avulsions between 1936 and 1941, but such phenomena are normal alluvial processes and are especially common on large deltas. They will not be equally effective every year in the future, but, in general, they will occur with increasing frequency and their effect will extend progressively farther upstream. Some of the sediment already deposited above the reservoir certainly will be moved downstream by floods from time to time; but, in general, the proportion of Rio Grande sediment deposited above the reservoir will certainly increase as more of the delta is built above reservoir pool elevation. Mr. Stevens suggests that increased runoff may counterbalance the tendency toward greater deposition above the reservoir, but there is no certainty either of increased runoff or of greater sediment production if the runoff does increase. Most of the Rio Grande water is derived from winter precipitation in the mountains of northern New Mexico and southern Colorado, which contribute relatively little sediment. The summer flash floods on intermittent tributaries in central New Mexico contribute most of the sediment, concentrations as high as 68% by weight having been reached in suspended load samples from the Rio Puerco in 1941; but these summer floods contribute very little of the total annual runoff. Under these conditions an increase in total runoff obviously might not involve any increase in sediment production, and might even be accompanied by a decrease in the total sediment load.

In contrast to such uncertain factors as future changes in runoff, and their effect on sediment transportation, the only question concerning delta growth is the magnitude of its effect. The data now available afford a reasonable basis for estimating this effect. If the 1914–1935 and the 1935–1940 rates of reservoir capacity loss are plotted as decreasing functions of the combined sedimentation in the reservoir and the 15 miles of valley affected by delta growth, and if a smooth curve is drawn through points representing the average values for these two periods, projection of this curve indicates that the "most useful life" of the reservoir as defined by Mr. Stevens, or the time until it will be half filled with sediment, will be about 90 years from the present, or nearly twice as long as that predicted by the straight-line projection. Further projection of the curve would indicate even greater increase in the total life of the reservoir, but projection so far into the future is of doubtful validity. The major importance of the factor of delta growth is fully emphasized by the indicated increase in the "most useful life."

Table 3 appears to show close agreement between suspended load records and the writer's estimates, based on surveys and mineralogical studies of the sediment contributions from five major Rio Grande tributaries above Elephant Butte Reservoir. The close agreement between these figures is fortuitous, however, and possibly misleading. The estimates of contribution to Rio Grande valley deposits should be nearly doubled if allowance is made for sediment carried into Elephant Butte Reservoir. The mineralogical data are not directly applicable to the sources of reservoir deposits, but reasonable

extrapolations from available data suggest that the average rate of sediment contribution from these tributaries should be about 2,500 tons per sq mile per year, instead of 1,500 which includes only that part deposited above Elephant Butte Reservoir.

The average contribution computed from suspended load measurements also is probably too low. Apparently this is based on Soil Conservation Service sampling from 1937 to 1941, which was too infrequent and too irregular to provide more than rough comparisons between different streams. The summer flash floods, which obviously carry most of the sediment, were not sampled sufficiently to provide a reliable measure of the total annual load, except possibly for the Rio Puerco during 1941. An average of 1,700 tons per sq mile would indicate that these major sediment contributing areas produced only about four times as much sediment, per unit area, as all the remainder of the Rio Grande drainage area above Elephant Butte, most of which obviously produces negligible quantities of sediment. Such a relation appears unreasonable, in the light of all other evidence; but the suspended load records are not adequate to determine whether or not the rate should be as high as 2,500 tons per sq mile, as indicated by the surveys and mineralogical studies.

It does not necessarily follow that the contribution from direct tributaries to Elephant Butte Reservoir and Lake Mead should be estimated as high as 2,500 tons per sq mile, or that the average specific weight of 65 lb per cu ft for Elephant Butte deposits, as derived by Mr. Stevens, should be raised accordingly. The nature of the terrain suggests, on the contrary, that 1,600 tons per sq mile may be more reasonable for these areas than the true figure, whatever it may be, for the five major Rio Grande tributaries above Elephant Butte. However, there is a serious sediment problem in the Rio Grande valley above Elephant Butte Reservoir, and remedial measures are urgently needed there. It is important that the major sources of Rio Grande sediment be correctly evaluated, within reasonable limits, in order that remedial measures may be wisely concentrated where they will be most effective. Mr. Stevens has recognized that the rates based on surveys and mineralogical studies, as listed in Table 3, do not represent total sediment output, but he has not indicated the magnitude of variance. It should be understood that those rates are probably only about half the actual average rates of sediment contribution to the Rio Grande from the Rio Puerco, Rio Salado, Jemez Creek, and Galisteo Creek.

Data are not yet available for quantitative estimate of the effect of delta growth on the rate of capacity loss in Lake Mead; but there is no apparent reason why the same principles will not apply there as in Elephant Butte Reservoir, although the magnitude of effect may be less.

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DISCUSSIONS

THE MECHANISM OF ENERGY LOSS IN FLUID FRICTION

Discussion

BY HUNTER ROUSE AND A. A. KALINSKE

HUNTER ROUSE,[41] M. AM. SOC. C. E., AND A. A. KALINSKE,[42] ASSOC. M. AM. SOC. C. E.[42a]—Energy dissipation in turbulent flow is at best an extremely complex phenomenon, and a successful investigation of the mechanics of such a process requires more ability, intuition, and courage on the part of the investigator than perhaps any other phase of fluid motion. It is for this reason that the writers have only commendation for the authors of the present paper in undertaking so broad and fundamental a treatment of the problem. No fault can be found with the mathematical derivations, and portions of the interpretive matter are considered well worthy of reading and rereading by every one interested in fluid mechanics. Section 10, for instance, which contains a detailed description of the turbulence mechanism, is particularly lucid and significant.

Because of the very complexity of fluid turbulence, a paper of this nature can be of value to only a limited number of readers if the treatment is not in itself a simple one, devoid of all ambiguous or superfluous material. The authors presumably sought to clarify the problem for more than such a limited group, through use of analogies, new concepts, and numerous charts; but the writers are inclined to question whether the goal of clarity has actually been achieved. Indeed, since the essential justification of the new concepts introduced by the authors was not properly noted, the application of these concepts to laminar and turbulent flow alike served to conceal rather than to emphasize the fundamental difference between the two types of motion.

Perhaps the writers' thesis may best be explained by reverting to the authors' use of analogy, to emphasize the fact that analogies are, to say the least, frequently misleading in their oversimplification. Instead of the pulley

NOTE.—This paper by Boris A. Bakhmeteff and William Allan was published in February, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1945, by George R. Rich, J. C. Stevens, and Leon Beskin.

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[42a] Received by the Secretary October 3, 1945.

system to illustrate the general flow process, one might more logically consider a uniform pile of lumber. If an equal force is applied to the end of each plank to simulate a constant pressure gradient, movement apparently comparable to that of laminar flow will result in the longitudinal direction. Thus, assuming a constant coefficient of friction (that is, "viscosity") between planks, the intensity of shear will increase linearly with depth of pile, just as—although for a different reason—the shear increases linearly with distance from the water surface or the center line of a pipe. The work done by the force acting on the end of the top plank, therefore, will evidently not be equal to the generation of thermal energy through friction with the next plank, since both planks are moving and a portion of the force will be transmitted via shear to the lumber below. However, to assume that the forces exerted upon the plank ends represent a form of potential energy (just as the pressure within a fluid is often improperly assumed to be potential energy of flow), which is then "transferred" by the shear process from one plank to another until finally dissipated in the form of heat, is hardly in accordance with accepted methods of mechanics. Moreover, it obviously would be impossible to extend either the plank analogy or the pulley analogy to the case of turbulent motion, since there is no conceivable intermixing process that the planks could undergo to simulate the well-recognized mechanism of turbulent diffusion. Neither analogy (nor, for that matter, any other) is thus capable of truly elucidating the problem under consideration.

In order to be rigorously acceptable in the mechanics sense, the authors' concept of energy borrowing and transmission must involve an actual mass exchange between neighboring zones of flow. Such an exchange is the very essence of turbulent motion, to be sure, but it appears at first glance to be no more characteristic of laminar motion than of the plank analogy. It must be recalled, nevertheless, that the large-scale or molar transfer of fluid matter produced by the mixing process of turbulence has its counterpart in the infinitesimal molecular mixing which takes place in turbulent and laminar flow alike. As a matter of fact, the kinematic molecular viscosity ν and the kinematic eddy viscosity ϵ are both products of mixing lengths and velocities, differing effectively only in scale. Just as the mixing process of turbulence is considered to produce a shear between neighboring zones of flow which is proportional to the lateral rate of momentum transfer, the viscous shear of laminar motion may be resolved into the lateral transfer of momentum through mixing on a molecular scale. Indeed, one frequently compares the rates of molecular and molar diffusion of salinity, or of heat, so that the molecular transfer of kinetic energy is actually just as real a process in laminar flow as the molar transfer is in turbulent flow.

The authors' concepts of energy transmission in both laminar and turbulent flow are therefore fully justifiable, although for reasons not brought to the attention of the reader. In the light of this justification, however, the essential difference between the two types of flow still must be properly clarified. The writers seek to accomplish the necessary distinction by assembling certain of the curves in the paper, together with additional functions which they have discussed only casually, in the schematic chart of Fig. 31.

In the uppermost diagrams are included the primary characteristics of laminar and turbulent flow in pipes: (1) The pressure gradient, which is constant across both sections; (2) the intensity of shear, which increases linearly with the radius in both cases; and (3) the velocity, which varies parabolically in laminar flow, and approximately logarithmically in turbulent flow. Neither these nor the remaining curves, it must be emphasized, are drawn to a particular scale, since it is the form of each function, rather than its absolute location, which is essential to the argument.

The diagrams next in order depict the mixing characteristics of the secondary motion—molecular in the laminar case and molar in the turbulent. Curve 4(*c*) represents the mean molecular velocity u'_m, and 4(*d*) the mean velocity $\sqrt{\overline{(u')^2}_t}$ of the turbulence (to which $\sqrt{\overline{(v')^2}}$ is assumed proportional); curve 4(*c*) is necessarily constant across the section, whereas curve 4(*d*) decreases markedly toward the center line. Curve 5(*c*) represents the mean free path l_m of the molecules, which is likewise constant across the section; but its molar counterpart curve 5(*d*), the mixing length or mean eddy size l, increases toward the center line. The remaining curves 6(*c*) and 6(*d*) for the mixing coefficients ν and ϵ are simply the products of the velocity and length characteristics of the respective mixing processes. Noteworthy is the fact that ν is constant across the section while ϵ reaches a maximum approxomately midway between center line and wall. In either case, nevertheless, the mixing functions are related to the shear and velocity curves of the upper diagrams through the parallel expressions $\tau = \nu\,\rho\,\frac{du}{dy}$ and $\bar{\tau} = (\nu + \epsilon)\,\rho\,\frac{d\bar{u}}{dy}$.

The third pair of schematic diagrams portray the transfer and the spending of kinetic energy by the mean laminar and turbulent motion. The local rate of spending, rather than the cumulative emphasized by the authors, is used herein for the same reason that local conditions of velocity and shear rather than their cumulative effects are considered more significant in the present argument. Likewise, instead of the authors' cumulative transmission function W_t, the writers have chosen the ratio $W_t/2\,\pi\,r$, since it is the latter quantity which is directly proportional to the mixing coefficient and the velocity gradient of the foregoing diagrams. In other words, the rates of transfer of kinetic energy of the mean motion toward the walls are, in terms of the respective mixing processes: $\left(\frac{W_t}{2\,\pi\,r}\right)_{\text{lam}} = \nu\,\frac{d\left(\frac{\rho\,u^2}{2}\right)}{dy}$ and $\left(\frac{W_t}{2\,\pi\,r}\right)_{\text{turb}} = (\nu + \epsilon)\,\frac{d\left(\frac{\rho\,\bar{u}^2}{2}\right)}{dy}$.

Although it is evident from these diagrams that both the transfer and the spending of the energy of the mean motion in turbulent flow are greater in the wall region, owing to the different nature of the mixing process, the true significance of these curves becomes apparent only through their correlation with the fourth pair of diagrams. The spending curve 6(*e*) for laminar flow, it must be noted, represents essentially direct dissipation into heat. The corresponding spending curve 6(*f*) for turbulent flow, on the contrary, embodies not only direct dissipation but also the conversion of energy of the mean motion into energy of turbulence through the continuous generation of new

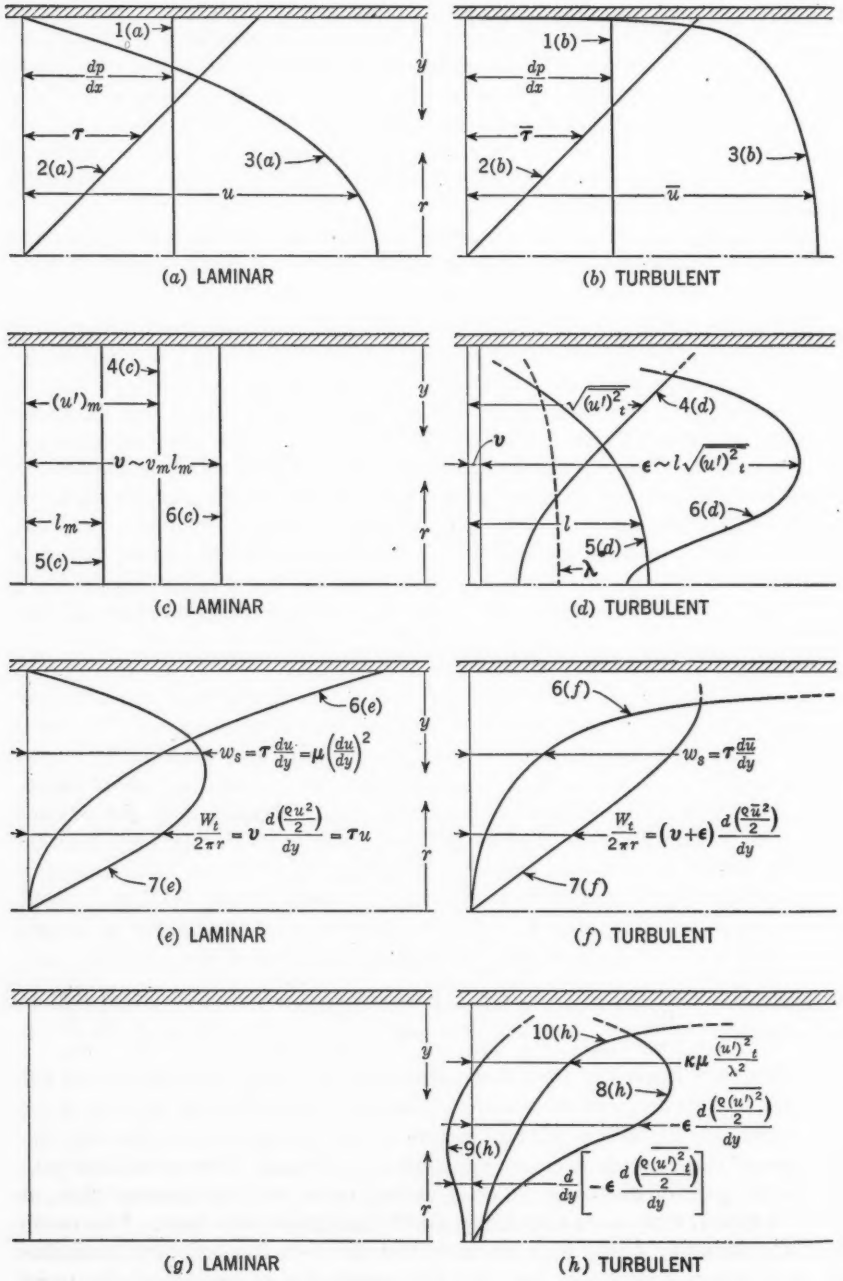

FIG. 31.—COMPARISON OF CHARACTERISTIC FUNCTIONS FOR LAMINAR AND TURBULENT FLOW IN PIPES

eddies.[43] As these eddies are formed, they (and their kinetic energy) become subject to the same lateral transfer as has already been discussed for the mean motion. In the case of laminar flow, the constancy of both the mixing coefficient ν and the kinetic energy $\frac{\rho(u')^2_m}{2}$ of the molecular motion results in a zero rate of transfer, diagram (g) therefore remaining blank. The rate of transfer of the kinetic energy of the turbulence, however, will be both finite and variable across the section, since—as shown by curve 8(h)—it must equal the product of the mixing coefficient ϵ and the negative gradient of $\frac{\rho\overline{(u')^2}_t}{2}$. Such transfer, in contradistinction to that of the kinetic energy of the mean flow 7(f), will be directed toward the center line, since that is the direction of decreasing $\overline{(u')^2}_t$.

The gradient of this transfer function (that is, the difference between the rates of efflux and influx of energy for a unit volume), plotted as curve 9(h), represents the difference between the local rate of turbulence generation and the local rate of turbulence decay. Near the wall, where the curve is positive, the generation process evidently predominates, but over the major part of the central region negative values indicate preponderant decay. The local dissipation rate, in turn, may be evaluated independently in the approximate form $\frac{\kappa\,\mu\overline{(u')^2}}{\lambda^2}$ noted by the authors and plotted as curve 10(h). (The length λ, it must be remarked, differs from the mixing length or eddy size l in that it represents the average scale of the smaller eddies which control the dissipation process; its trend is indicated in diagram (d) by a broken line.) Comparison of the dissipation curve 10(h) with the spending curve 6(f) will now show that the two differ essentially by the local magnitude of curve 9(h), which is positive near the walls and negative near the center. In other words, if the comparatively small rate of dissipation by viscous action in the mean motion (not plotted herein) is ignored, the spending curve 6(f) will be seen to represent the algebraic sum of the turbulence-dissipation curve 10(h) and the generation-minus-decay curve 9(h), in accordance with previous statements. Thus the ultimate difference between the laminar and the turbulent mixing processes lies in the fact that turbulence generated near the walls is continuously being transferred toward the center, so that the dissipation rate in this region actually exceeds the rate of spending.[44]

Despite the qualitative clarity which such an analysis should yield, the fact must be stressed that existing experimental data permit exact evaluation of only part of the functions shown schematically in the several diagrams. The laminar curves, of course, are accurately known. Completely lacking, however, are systematic measurements of $\overline{(u')^2}_t$, l, λ, and ϵ, for various boundary and flow conditions. Even the curve of velocity distribution in turbulent flow becomes indeterminate in the neighborhood of rough walls, as a result of

[43] "Statistical Theory of Turbulence," by Theodor von Kármán, *Journal of the Aeronautical Sciences*, Vol. 4, No. 4, 1937, p. 135.

[44] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Series A, Vol. CLI, 1935, pp. 462–464.

which all curves for turbulent flow in Fig. 31 have advisedly been broken in the wall vicinity. Just what happens in the neighborhood of the wall, and at the wall itself, must therefore remain for the present a matter of complete conjecture. It is possible, however, to perform the following bulk evaluation of the relative parts played by the wall and the central regions, which may shed further light upon the dissipation problem.

In the case of smooth pipes, of course, the method of analysis discussed in the foregoing pages can be carried to the wall itself, for reasonably precise information as to the velocity distribution from boundary to center line is at hand. If one were to assume, arbitrarily, that the nominal thickness of the laminar sublayer $\delta = \frac{11.6\,\nu}{\sqrt{\tau_o/\rho}}$ represents[45] the border between the central and the wall regions, it becomes possible at once to evaluate independently W_c, W_w, and W_o, the rates of energy spending per unit length of pipe in the central region, the wall region, and over the entire cross section, respectively. For example, since the velocity gradient in the sublayer is very nearly constant, as a close approximation (except at very low Reynolds numbers) the rate of dissipation in this wall zone will be $2\,\pi\,r\,\delta\,\mu\,(du/dy)^2$, which reduces to

$$W_w = 11.6\,\pi\,D\,\tau_o\,u_f \qquad (101a)$$

while the total rate of energy expenditure for the pipe becomes, in similar units,

$$W_o = \pi\,D\,\tau_o\,u_f/\sqrt{f/8} \qquad (101b)$$

The following ratio then results:

$$\frac{W_w}{W_o} = 11.6\sqrt{\frac{f}{8}} \qquad (102)$$

Since the resistance coefficient f is a known function of the Reynolds number, a plot of W_w/W_o versus **R** may be made directly.

A diagram of a somewhat similar nature[46] is reproduced in Fig. 32. The over-all ordinate AD of this diagram represents the total rate of energy expenditure as given by Eq. 101*b*, and the ordinate AB the rate of dissipation within the laminar sublayer approximately in accordance with Eq. 101*a*. The spending rate in the central region, shown by the ordinate remainder BD, has been further subdivided into part BC, the rate of dissipation due to the mean motion, and part CD, the rate of dissipation within the superposed turbulence. Immediately evident is the fact that the wall region plays a relatively large part in the total dissipation process, although the turbulence mechanism becomes more and more effective with increasing Reynolds number (that is, decreasing thickness of the laminar sublayer). Equally apparent is the very

[45] "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 498.

[46] "Sur la répartition entre le mouvement moyen et le mouvement d'agitation de l'énergie dissipée dans l'écoulement turbulent d'un fluide incompressible," by J. Kampé de Feriet and A. Martinot-Lagarde, *Comptes rendus des seances de l'Académie des Sciences*, December 13, 1937.

small rate of dissipation attributable to the mean motion in the central region, which likewise diminishes with increasing **R**.

Comparable representation of the relative dissipation rates for rough pipes would require a separate diagram for each successive degree of roughness. It

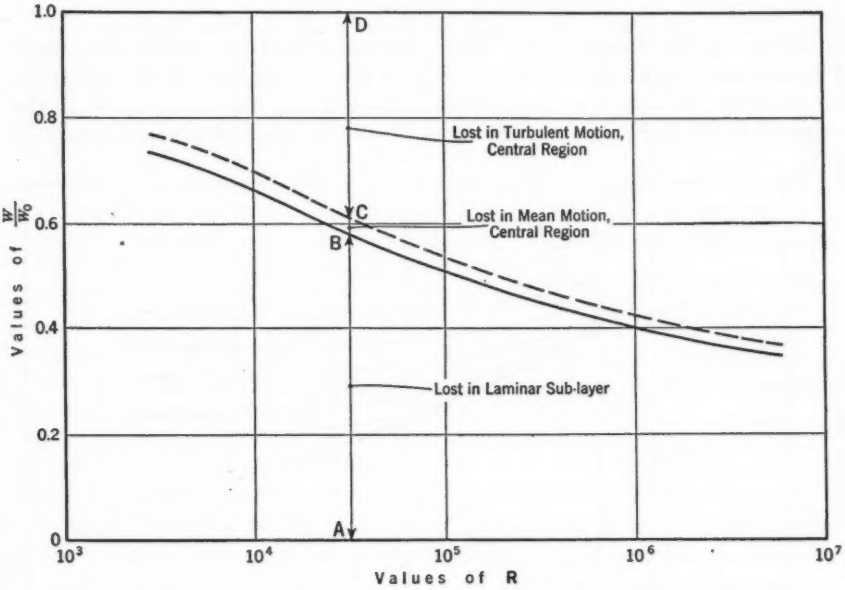

FIG. 32.—RELATIVE RATES OF ENERGY LOSS IN DIFFERENT ZONES OF FLOW FOR SMOOTH PIPES, AFTER KAMPÉ DE FERIET AND MARTINOT-LAGARDE

would appear, moreover, that preparation of such diagrams must entail equally explicit information as to flow conditions near the boundary, which information is notably lacking in the case of boundary roughness. However, it will be recalled that the ratio W_w/W_o is equal to $1 - W_c/W_o$, which can be evaluated independently for all conditions. The authors, for instance, utilize this fact in interpreting the similar gradients of the velocity curves for different roughnesses in Fig. 16. That is, by assuming the same hydraulic slope for flow through pipes of the same diameter but different relative roughness, they show that the energy expenditure in the central region will be identical, whereas the total expenditure—and hence that in the wall region—will decrease with increasing roughness. The writers believe, however, that a far more general and significant interpretation may be made.

As must be concluded from Fig. 16, the dimensionless gradient $\frac{d(\bar{u}/u_f)}{d(r/r_o)}$ is the same for all velocity-distribution curves at any particular value of r/r_o in the central zone, regardless of the magnitude of the Reynolds number or the relative roughness. Therefore, the rate of energy expenditure in this central zone, up to that value of r/r_o at which the type of boundary begins to

influence the given dimensionless gradient, may be expressed in completely general terms. In other words, letting $x = \bar{u}/u_f$ and $z = r/r_o$, it may be shown that

$$W_c = \pi D \tau_o u_f \int_0^{z'} z^2 \frac{dx}{dz} dz \quad \ldots\ldots\ldots\ldots\ldots\ldots (103)$$

in which z' is the limiting value of r/r_o for the central zone under discussion. The integral may readily be evaluated from the logarithmic velocity equation, for which $dx/dz = 1/(1 - z)$, its magnitude evidently depending solely on the value of the limit z'. Upon division of Eq. 103 by Eq. 101*b*, and subtraction

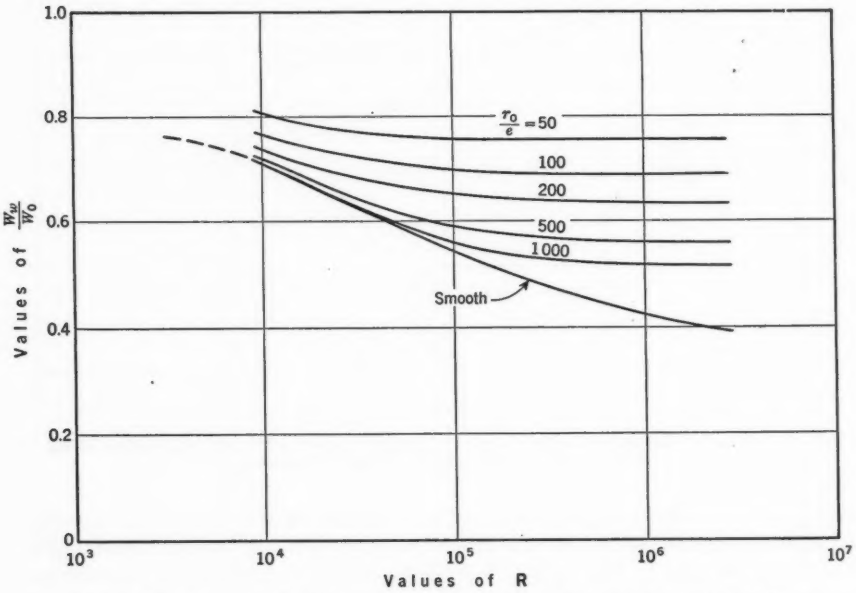

FIG. 33.—RELATIVE RATES OF ENERGY LOSS IN WALL REGION FOR SMOOTH AND ROUGH PIPES

from unity, the desired ratio of the rates of wall and total energy expenditure is seen to become simply a function of z' and f:

$$\frac{W_w}{W_o} = 1 - \frac{W_c}{W_o} = 1 - \sqrt{\frac{f}{8}} \int_0^{z'} z^2 \frac{dx}{dz} dz \quad \ldots\ldots\ldots\ldots (104)$$

The question still must be answered as to what magnitude of z' to specify in determining values of W_w/W_o to plot against **R** and e/r_o. In the case of smooth pipes, of course, use of $z' = 1 - \frac{\delta}{r_o}$ will reduce the result to that of Eq. 102. In the case of rough pipes, it would then appear reasonable to seek a function of the roughness e which would involve a comparable portion of the pipe section; it would be desirable, moreover, to incorporate in this function

the effect of viscosity in the practically important transition range from the smooth to the rough phase of the resistance phenomenon. Comparison of the intercepts on the smooth-pipe and the rough-pipe velocity-distribution curves indicates that a value of $3\,e$ is approximately the counterpart of the value δ used in Eq. 101*a*; and further consideration indicates that the relationship $1 - z' = \dfrac{\delta + 3\,e}{r_o}$ is the only simple function approaching the assumed limits of $\dfrac{\delta}{r_o}$ and $\dfrac{3\,e}{r_o}$.

Based upon this function, the writers have prepared from Eq. 104, together with a diagram of f versus **R** for smooth and rough commercial pipes,[47] the system of curves of W_w/W_o versus **R** shown in Fig. 33. Despite the admittedly arbitrary nature of the assumed relationship for z', it is believed that this diagram gives a particularly clear picture of the part played by the wall region in the process of energy expenditure, and of its variation with the relative roughness and the Reynolds number. In fact, only through a plot of this nature is it truly possible to judge the effect of roughness under comparable flow conditions. The similarity of Fig. 33 to the f : **R** diagram is simply an indication of the close relationship which exists between f and the ratio W_w/W_o in the several equations given.

If the writers' premise that Eq. 103 reduces the mechanism of energy transfer and expenditure in the central region to a truly common denominator is justified, it must be concluded from Fig. 33 that the rôle played by the wall zone increases markedly with the relative roughness under comparable flow conditions (that is, constant Reynolds number rather than constant hydraulic slope). It must be realized, of course, that the relative thickness of the wall zone considered by the writers is by no means fixed, since $1 - z'$ has been assumed proportional to both δ/r_o and e/r_o, which vary considerably in magnitude. On the other hand, since the velocity curve in the central region depends upon these same two variables, and hence is just as flexible a function, the writers' assumption appears in close accord with the probability that the zone of marked turbulence generation must also vary in radial extent as the Reynolds number and the relative roughness change.

Since, in the case of fully developed roughness action, there is no longer a laminar film at the wall in which direct dissipation of energy can occur, one is also led to conclude that the increase in the proportional rate of energy expenditure in the wall region must be due to the relative violence of the turbulence generated by the wall irregularities. It must not be assumed, however, that this large rate of expenditure represents an equally large rate of conversion into energy of turbulence which is subject to transfer toward the central region. On the contrary, as shown by the authors, a considerable portion of the actual energy dissipation probably occurs locally during the generation process. This is due to the extremely high instantaneous stresses produced within the eddies as they are formed. The energy required to maintain equilibrium of the turbulence structure, in other words, while eventually

[47] "Evaluation of Boundary Roughness," by Hunter Rouse, *Proceedings*, 2d Hydr. Conference, June 1-4, 1942, State Univ. of Iowa, Iowa City, *Bulletin No. 27*, Studies in Eng., 1943.

dissipated in its entirety, does not all pass through the intermediate stage of transferable energy of turbulence.

In this discussion the writers have consistently sought to add to, rather than detract from, the value of the authors' detailed presentation. As a new approach to a problem of vital importance is invariably subject to modification and improvement, the writers are hopeful that the interpretation presented herewith may likewise be altered and extended by the authors in their closure, and by those who later use the paper as a reference. Indeed, quite aside from the fundamental material which the paper has provided, it is probable that it will have lasting worth as a stimulus to creative thought on a very stubborn problem.

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DISCUSSIONS

MOVING LOADS ON RESTRAINED BEAMS AND FRAMES

Discussion

BY MAX W. STRAUSS, D. A. MACKENZIE, AND CHARLES E. SCHAFFNER

MAX W. STRAUSS,[14] JUN. AM. SOC. C. E.[14a]—Publication of this paper at long last makes available to the American engineering profession a method for the precise analysis of rigid frames that heretofore has been widely used in other countries. (A relatively small group in the United States, of which the writer is a member, had the privilege of studying under Professor Brumfield and thus was aware of this method prior to its present publication.)

The concept of an exact "carry-over factor" or "transmission coefficient" is not a new one. It can be traced back to the famous French engineer, Maurice Lévy, in whose great work[15] may be found the equivalent of many of the author's expressions. In the United States the concept was introduced in graphical form[16] by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., as the method of conjugate points. More recently an analytic application of this concept was implied by Camillo Weiss,[17] M. Am. Soc. C. E. In his paper, Professor Brumfield presents a logical development of this analytic application.

Restraining Moments in a Loaded Span.—In Sections 4 and 5 of the paper, the author has derived expressions for the end moments in a loaded span due to the loads in that span. Eqs. 14, although quite simple, are limited in use to concentrated loads. For other loads or combinations of loads, the labor involved becomes somewhat tedious in that the analyst may either integrate Eqs. 14 (mathematically, if the particular loading so allows, as in Eq. 15, or otherwise mechanically), or use Eqs. 27 in conjunction with the curves of Fig. 5,

NOTE.—This paper by R. C. Brumfield was published in May, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1945, by R. W. Stewart, Nathan D. Brodkin, and T. F. Hickerson.

[14] Asst. Structural Engr., Dept. of Water Power, Los Angeles, Calif.

[14a] Received by the Secretary August 27, 1945.

[15] "La Statique Graphique et Les Applications Aux Constructions," by Maurice Lévy, Gauthier-Villars & Co., Paris, France, 1886–1888.

[16] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 1.

[17] "Interrelation of Certain Structural Concepts," by Camillo Weiss, *Proceedings*, Am. Soc. C. E., January, 1945, p. 63.

Eq. 22*b*, and Eq. 24. However, much of this tedious labor can be avoided by a single expression that is applicable to any load or combinations of loads.

Eq. 14*b* may be rewritten—

$$M_R = -WL\frac{C_R}{1-C_R C_L}[(2k^2-2k^3+k-2k^2+k^3) - C_L(2k-4k^2+2k^3+k^2-k^3)] \quad (55)$$

Remembering that the fixed-end moments, M_{FR} and M_{FL}, for the right end and left end, respectively, for a concentrated load are given by

$$M_{FR} = (k^2-k^3)\,WL \quad (56a)$$

and

$$M_{FL} = (k-2k^2+k^3)\,WL \quad (56b)$$

and substituting these in Eq. 55, the latter becomes

$$M_R = -\frac{C_R}{1-C_R C_L}[(2M_{FR}+M_{FL}) - C_L(2M_{FL}+M_{FR})] \quad (57a)$$

Although Eq. 57*a* is adequate, it can be simplified further. By collecting terms:

$$M_R = -\left[M_{FR}\frac{C_R(2-C_L)}{1-C_R C_L} + M_{FL}\frac{C_R(1-2C_L)}{1-C_R C_L}\right] \quad (57b)$$

Substituting Eq. 4 for C_R and C_L, combining like terms, and then substituting Eq. 40, Eq. 57*b* reduces to

$$M_R = -[M_{FR}(1-t_R) + M_{FL}t_L C_R] \quad (58a)$$

and, similarly,

$$M_L = -[M_{FL}(1-t_L) + M_{FR}t_R C_L] \quad (58b)$$

in which t_R and t_L are obtained by Eq. 41 or from Col. 10 of Table 1.

Eqs. 58 have the advantage of being applicable to all types of loadings inasmuch as tables and curves for fixed-end moments for all types of loads are readily accessible in all standard reference books.

Stiffness and Restraint.—It seems to the writer that I/L, or K, affords a better mental image of the stiffness of a span than the author's L/I, or Q, since a stiffer beam is thus represented by a larger numerical value. Using larger numbers for stiffer beams avoids the mental juggling that must inevitably ensue if the reverse is adopted. Thus

$$i = \frac{2-C}{K} \quad (59)$$

and if, furthermore, the stiffness index is defined as $1/i$ or r, instead of i, and the restraint index is defined as the sum of the stiffness indexes, then

$$r = \frac{1}{i} \quad (60a)$$

$$R' = \frac{1}{R} = Sr \quad (60b)$$

and

$$C = \frac{1}{2+\frac{K}{R'}} \quad (60c)$$

This is of more than academic interest since it would simplify the necessary calculations considerably. For example, if Col. 3 of Table 1 listed r, or $1/i$, instead of i, then Col. 4, which would list R' instead of R, would be obtained merely by the addition of the values in Col. 3 for the other members at that joint. The computer would not be required to obtain the reciprocal of the sum of the reciprocals of these values.

Transforming the author's expressions so as to utilize fixed-end moments serves the further purpose of illustrating the coordination of this method with the Cross method of successive approximations and the slope-deflection method. This is to be expected as all are based on, and derived from, the basic theorem of three moments.

D. A. MACKENZIE,[18] Assoc. M. Am. Soc. C. E.[18a]—The analysis of continuous beams and frames supporting moving loads has been advanced considerably by the publication of this paper. Eqs. 29 and the curves of Fig. 6 are particularly useful. A precise mathematical formula now is available for locating the

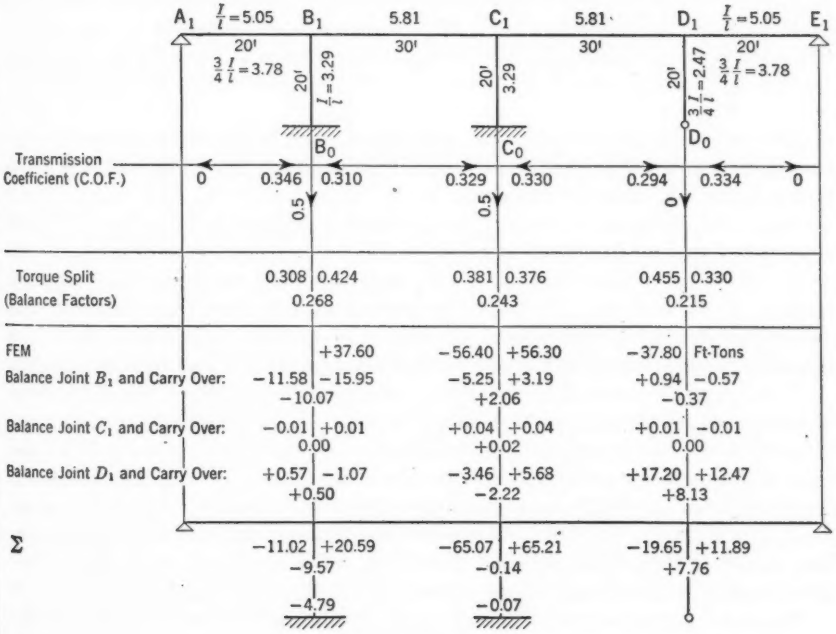

FIG. 15

loads to produce maximum moments, instead of the more or less approximate methods commonly used in the past.

The coefficients used for transmission (C), moment split (m), and torque split (t) may also be used as moment distribution factors and, once the loads

[18] Design Engr., Buffalo-Niagara Elec. Corp., Buffalo, N. Y.

[18a] Received by the Secretary September 13, 1945.

are located on the span, the negative moments may then be found by fixed-end moments and direct moment distribution.

The author's example of Fig. 12 may be arranged as shown in Fig. 15 and the end moments readily found by that method.

Joint B_1 is first balanced to make $\Sigma M_{B1} = 0$. The moment (15.95) allocated to B_1C_1 is then multiplied by the carry-over factor ($C = 0.329$) to produce a moment of 5.25 ft-tons at joint C_1 which is then distributed to the other two members in proportion to their torque-split factor (t) or their moment-split factor (m), and the signs changed to make $\Sigma M_{C1} = 0$. These are carried over progressively to the other joints in the same manner; and the other joints are then balanced likewise.

The sign convention used is one which considers that a moment at the end of a member is positive if it tends to rotate the adjacent joint clockwise.

The computations in this discussion were done by slide rule, which probably accounts for the slight difference from the author's values.

Of course, the moments could also be found by the Hardy Cross method, but, having already obtained the direct-moment factors, the amount of work involved in making the balancing and distributing computations is very small. The principal advantage of the direct method is its flexibility for determining maximum moments where some spans are loaded and others not loaded.

CHARLES E. SCHAFFNER,[19] JUN. AM. SOC. C. E.[19a]—The most serious handicap of the Brumfield method is that it is comparatively difficult to learn without the help of an instructor.

One point concerning the paper, as published, is open to criticism. This is that it was begun with complete definitions of the special nomenclature to be used. Because of the newness of the approach, it is necessary to use unfamiliar nomenclature. However, psychologically, it might be better if the author had defined the terms and symbols as needed, throughout the text, and then summarized them at the end of the paper, or at the end of each section.

The only familiar symbol that has been changed is the Q-value which is the reciprocal of the familiar "K-value." It is well that this change has been made because, for this particular method, it is more advantageous to have the value in the numerator than in the denominator. The algebraic manipulations are simplified. In addition, it must be remembered that all writing, derivations, and solutions are done initially by hand rather than by the typewriter. The use of Q thus eliminates the possibility of confusing K with the k frequently used to denote a proportionate part of the span length.

The actual simplicity of the method, once one is familiar with the fundamentals, will be apparent upon examination of Eqs. 14, 16, and 17, which are derived in straightforward fashion from the theorem of three moments. The author has verified the accuracy of each by simple examples.

Many engineers have a phobia against using any equations for which curves are plotted. The curves, shown in Figs. 5 and 6, aid in the solutions of Eqs. 26 and 29. However, they are not indispensable, as the equations can be

[19] Instructor, Civ. Eng. Dept., Cooper Union, New York, N. Y. (on military leave).

[19a] Received by the Secretary September 26, 1945.

solved easily should the curves be unavailable. In fact, as the author notes, Eqs. 25 need not be used. The separate effects of a series of concentrations may be computed by the use of Eqs. 14, 16, or 17, and these effects can be added algebraically. However, the computing time is reduced considerably by the use of Eqs. 25; and it is reduced much more if Eqs. 27 and Fig. 5 are used.

In Section 7, the author shows good judgment by explaining the significance of a number of the new terms. One of the faults inherent in most involved systems of computation is that the computer gradually tends to do things by rote, and thus loses understanding of the different steps through which he passes. These terms are not just invented terms, but have a definite physical meaning. Thorough understanding of their use and significance is essential.

In Sections 6 and 8, the author succeeds where the proponents of all other methods have failed. He has shown how to locate the position of a moving load on a restrained beam for maximum bending moment, either at the support, or under one of the concentrations. This will provide a great reduction in the work and time necessary in analyzing this type of problem.

The derived equations, although not simple, can be handled competently, as the author shows in his illustrative example in Section 6. It is regrettable that no example of the use of Eq. 35 has been given, and that some of the steps in its development have been omitted. However, it is obvious that in a short paper, such as this, some things, ordinarily included, must be omitted. Thus some excellent algebraic manipulations have been eliminated from this treatment. One idea which tends to inspire confidence in the equations is the manner in which the author shows how each equation reduces to a well-known formula under simple beam conditions. Actually then, related simple beam formulas are only special cases of the more general formulas developed in this paper.

It is imperative to become familiar with the frame characteristics, as explained in Section 10. These new terms—transmission coefficient, stiffness index, restraint index, moment split, and torque split—and their interrelations are the basis of the method. A complete understanding of this material is essential to the proper use of the method. The arrangement, shown in Table 1, includes all the characteristics of the frame that are necessary for the analysis. In addition, the value of the table is enhanced by its self-checking features which eliminate practically all chances of error.

In the larger, more complicated frames, it is impossible to complete this table without resorting to the estimation of the value of a few of the coefficients. This operation (see Section 11), although quite simple, is invaluable. It enables the analyst to check key members rapidly without conducting a complete analysis.

One of the most frequent sources of error and confusion in the analysis of rigid frames has always been the sign or sense of the bending moment. The traditional "positive or negative" of courses in mechanics of materials is usually abandoned because of the confusion created by other than horizontal members. Directional signs or the terms, "clockwise or counterclockwise," usually are adopted; and actually that is what the author's "normal and reverse system"

is. However, his concept, as given by his definitions, "pins down" the exact meaning of these terms, thus clarifying the physical picture in the mind of the user. These definitions, together with the other fundamentals, enable the analyst to picture exactly how each member of the frame reacts to different loading conditions. This gives the analyst a tremendous advantage over one who uses a system of positive and negative bending moment without attempting to picture the resultant bending of the frame.

In Section 13, Professor Brumfield mentions a moment-distribution chart for each joint. This is an excellent bookkeeping device. It must not be misconstrued, however, as an essential part of the method. Direct moment distribution, although a little cumbersome in the analysis of large frames, is no more so than that in the Hardy Cross system. The charts are something that the author has devised to facilitate computation. It has been the experience of the writer, working without these charts, that the method is easily adaptable to complicated reentrant frames. In one particular case it was used successfully on an airplane hangar upon which other men had worked, unsuccessfully, for much longer periods, with a number of other methods. When each of three other methods "bogged down" at a different spot, this method was finally tried and the analysis was completed in a surprisingly short period. As the author states, one distribution is sufficient for all purposes, including sidesway.

In conclusion, the author remarks that the method has been extended to beams of variable section and restrained ends, and also to curved beams. The writer has done some work in this respect in arch analysis. Probably the most popular method of arch analysis is the "elastic center method." A comparison of the two methods would therefore be important.

The "elastic center method" is derived from the laws governing elastic displacements in ribs and frames—a fairly involved subject. This method is based upon some of the fundamental concepts of the moment-area method, familiar to all students of elementary mechanics of materials. It is developed from these concepts, in direct fashion, mainly through the use of the laws of statics. Therefore, where the Brumfield method would seem to be comparatively difficult to learn as applied to straight beams, it becomes the simpler method when applied to the more complex subject of curved beams of variable section. Every equation used is transformed into the corresponding orthodox straight beam formula as the radius of curvature of the beam approaches infinity.

These same equations, applied to arches, sewer rings, etc., can be used with equal facility for the analysis of ordinary frames, such as rectangular bents, saw-tooth roof frames, etc. This means, of course, great versatility and adaptability. Finally, the Brumfield method reduces the arithmetical work necessary in the elastic center method by at least one third. This is of inestimable value, as it is this part of the analysis that presents the greatest chance of error.

To summarize, the writer believes that this method will gain rapidly in popularity among engineers when they realize that its accuracy, ease of usage, and extreme adaptability far outweigh the initial time spent in learning it.

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DISCUSSIONS

EFFECTS OF RADIANT HEAT ON REINFORCED-CONCRETE RIGID FRAMES

Discussion

BY N. M. NEWMARK, AND L. C. HOLLISTER

N. M. NEWMARK,[16] ASSOC. M. AM. SOC. C. E.[16a]—So much essential information has been omitted from this paper that it is difficult to follow the author's reasoning, particularly in those places where mistakes seem obvious.

The curves shown in Fig. 4 for the temperatures at various times in two different sections of the deck have the correct general shape, but the numerical values are grossly inconsistent. Since the temperature rise does not extend very far into the slab in either case, it seems unreasonable that the variation of temperature with depth should be different for the two sections. One should expect to have the same temperatures at equal depths, and, moreover, these temperatures should be the same as in an infinitely deep slab at the same depth from the surface. One suspects from the figure that the author used increments of depth of different lengths for the two sections, and obtained different curves by reason of inaccuracies in his numerical procedure. If this is the case, the lower set of curves should be more dependable, and should apply to all sections of the bridge.

The explanation of the computation of angle change in step 2 is not easy to follow. The writer does not understand why the relatively tedious computation of Table 1 is necessary when the angle changes can be written directly if one knows the area of the depth-temperature strain curve and the position of its centroid. Let the area of the depth-temperature strain curve be denoted by Q, and let the distance from the top of the slab to the centroid of the area be denoted by y_0. Then the angle change ϕ is given by the equation

$$\phi = \frac{Q\,(c - y_0)}{I} \qquad (16)$$

in which c is the distance from the top surface to the neutral axis of the section.

NOTE.—This paper by Milan A. Johnston was published in June, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1945, by I. Oesterblom.

[16] Research Prof., Civ. Eng., Univ. of Illinois, Urbana, Ill.

[16a] Received by the Secretary September 22, 1945.

In addition, the direct strain at the neutral axis, ϵ_c, from which the axial elongations may be determined, is given by the relation

$$\epsilon_c = \frac{Q}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which A is the area of the cross section. Values of Q and y_0 were estimated from the lower curve in Fig. 4(*b*) with some difficulty since the scale of the drawing is small. The results were: $Q = 8.9\ \alpha$ ft-degrees and $y_0 = 0.24$ ft. The straight-line distribution given on the figure as an equivalent strain distribution leads to slightly different results—namely, $Q = 8.0\ \alpha$ ft-degrees and $y_0 = 0.21$ ft.

The latter set of values is used in the further calculations below, where it is assumed that these quantities are the same for all sections, in contradistinction to the author's assumptions. Angle changes for sections 1, 5, and 9 are computed in Table 2. With $\alpha = 0.0000065$ ft per ft per degree, the value of Q becomes 0.000052 ft. It will be noted that only the angle change at section 9 checks the

TABLE 2.—ANGLE CHANGES FOR SECTIONS 1, 5, AND 9

Description	Section 9	Section 5	Section 1
Depth d, in ft	2.27	2.75	4.53
Moment of inertia I, in ft^4 per ft, $= \frac{d^3}{12}$	0.98	1.74	7.8
Distance to neutral axis $c = \frac{d}{2}$	1.14	1.38	2.27
$c - y_0$	0.93	1.17	2.06
Angle change ϕ, in radians per ft (Eq. 16)	0.000049	0.000035	0.000014
Angle change ϕ, in radians per in.	0.0000041	0.0000029	0.0000011
Strain ϵ_c at the neutral axis, in ft per ft (Eq. 17)	0.000023	0.000019	0.000011

author's value, since the author used different strain-distribution curves for the other sections. Incidentally, the units for angle change as given in the paper should be radians per inch. The author's diagram for angle-change loading is changed somewhat. The final values for fixed-end moments at B and C, estimated from the physical constants tabulated, are approximately +32.5 ft-kips instead of +37.5 ft-kips as given by the author (using the designer's convention where positive moment indicates compression in the top fibers).

The effect of the elongations at the neutral axis may be estimated as follows: The average strain is approximately 0.000018. This corresponds to an elongation of the neutral axis of 0.0014 ft. In order to bring the footings back to their proper position, rotations of joints B and D (Fig. 7) are required, of magnitude $\frac{0.0014}{15.84} = 0.00009$ radian. These correspond to fixed-end moments at B and C of magnitudes −7.4 ft-kips and +5.3 ft-kips, respectively, using the designer's convention.

With the author's numerical constants, the final moments in span BC are computed, as shown in Table 3. The final values are considerably less than those given by the author.

In interpreting these results, one factor completely neglected by the author must be taken into account. This factor is the stress due to the internal dis-

tortions caused by the nonuniform temperature. For example, if the deck were completely unrestrained at the ends, there would still be a statically balanced stress distribution at every cross section, corresponding to the conditions shown in Fig. 5(*b*). At section 9 the compressive strain at the top fiber,

TABLE 3.—FINAL TEMPERATURE MOMENTS, IN FOOT-KIPS (PLUS INDICATES COMPRESSION AT TOP FIBER)

Description	Point B of BC	Crown of BC	Point C of BC
Neglecting axial elongation	+28.7	+34.3	+35.2
Due to axial elongation	−8.5	−2.6	+4.7
Total moments	+20.2	+31.7	+39.9

even if no moment acted at the section, is determined from the quantities in Table 2, as follows: $\epsilon = 27\,\alpha - 0.000023 - 0.000049 \times 1.14 = 0.000176 - 0.000023 - 0.000056 = 0.000097$. The neglected compressive stress is, therefore, 291 lb per sq in. There is also a compressive strain at the bottom of the section, of magnitude 0.000079 in. per in., and tensile strains near the middle of the section. These stresses or strains are to be added to the others computed, and are of the same order of magnitude.

The net effect in this structure is almost, but not quite, the same as if the only stresses introduced by the radiant heat were the compressive stresses near the top of the deck required to eliminate the temperature strains near the top. This is not unreasonable because of the great restraint given the deck by the short stiff columns.

The foregoing comments are made in order to clarify the method of calculation which may perhaps find utility for other problems. However, the writer would like to emphasize that the stresses due to temperature should not be considered in the same light as stresses due to loads. Temperature strains are fixed by the temperature conditions, but the stresses produced thereby are reduced when the load stresses are high since the effective modulus of elasticity of the material is reduced for high stresses. In a structure made of concrete or of structural steel, temperature stresses are generally secondary stresses, and do not ordinarily greatly impair the capacity of the structure to carry overloads, although at working loads the stresses may be fairly high. The question raised by Mr. Johnston—whether the temperature effects can always be neglected—is an important one. Certainly in the structure he has considered, the radiant-heat effects do not appear to be serious.

L. C. HOLLISTER,[17] ASSOC. M. AM. SOC. C. E.[17a]—The effects of radiant heat on reinforced-concrete rigid frames, as outlined in the paper, are very interesting. Most engineers have been aware that, in some climates, the differential temperatures within a member of a concrete frame could be rather large. However, they have not bothered to attempt an analysis of such stresses; the paper, therefore, should be a valuable contribution.

[17] Engr. of Design, State Bridge Dept., Sacramento, Calif.

[17a] Received by the Secretary October 8, 1945.

Mr. Johnston finds, for the example used, that stresses due to radiant heat amount to 35.5% of the total design moment. This is astonishingly high, and indicates that designers should not completely ignore the possibility that such a condition exists.

The California State Highway Bridge Department has built many so-called rigid frame structures. It is the practice of this department to design

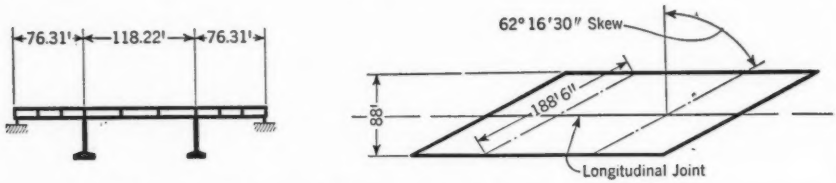

FIG. 8.—BRIDGE ACROSS THE ARROYO SECO CHANNEL NEAR HOUGH STREET, SOUTH PASADENA, CALIF.

for 35° temperature rise and 45° temperature drop in the colder parts of the state, and 30° rise and 40° drop in the more moderate areas. Stresses due to shrinkage, equivalent to about 30° drop in temperature, are also considered. Spans for the reinforced-concrete rigid frames built in California vary from moderate lengths to 130 ft. Some have quite wide roadways, and others have rather sharp skews, all of which tend to add to the intensity of temperature stresses. Some of these structures could be subjected to relatively severe radiant-heat stresses. For instance, the Sacramento and San Joaquin valleys of California have large daily temperature ranges throughout the summer months. The daily change in temperature is from 30° to 50° and a change as high as 62° has been noted in some parts of the state. With an accumulation of heat on the surface of the slab, to 25° or 30° above air temperature, it would not be unreasonable to expect the differential temperature from top of slab to bottom of girder to be around 60°. This is approximately twice that used in the "Numerical Example" of the paper. These structures are inspected frequently and no signs of distress that might be attributed to temperature have been noted.

Some modern framed highway structures are becoming so wide and so severely skewed that stresses due to temperature are becoming increasingly important. For instance, several years ago the state constructed a reinforced-concrete frame structure with the dimensions shown in Fig. 8. The superstructure was provided with a longitudinal joint down the center; but the two main piers were one solid mass of concrete 4 ft 9 in. wide by 188 ft 6 in. long. Inspection reveals no distress in the superstructure that could be blamed to temperature or radiant heat. The piers, however, have cracks which are probably due to shrinkage. Other than this there appears to be no harmful effects of any kind.

Another example is afforded by Fig. 9. To relieve temperature stresses it was necessary to provide a longitudinal joint down the center of the superstructure, and also a joint in the cap of the pier at the centerline. The bottoms of the columns are hinged to relieve temperature stresses.

It is regrettable that California does not have more specific data on the relative temperatures between top of slab and the interior parts of the girders, especially regarding some of the larger structures located in regions subject to high radiant heat.

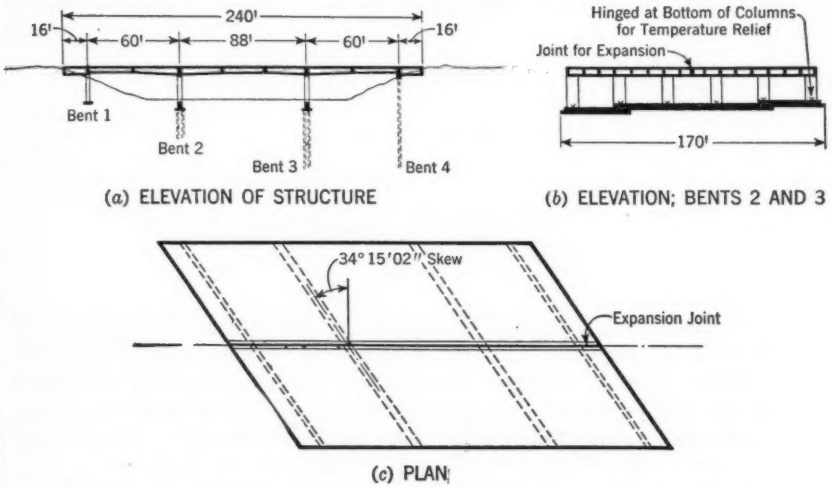

FIG. 9.—DESIGN OF UNDERCROSSING ON GLENDALE BOULEVARD, LOS ANGELES, CALIF.

The author has presented the paper in a clear, understandable manner, and outlines six well-defined steps necessary for the computation of radiant-heat stresses. The work of finding the angle changes at a given section would be somewhat more complicated for a T-beam or a box girder than for the example used; however, the same principle would apply.

Probably it will be a long time before designers are willing to go to the trouble of computing radiant-heat stresses as a regular practice. Nevertheless, it is well to have on record a method by which such stresses can be computed.

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DISCUSSIONS

STRUCTURAL SKEW PLATES

Discussion

BY JOHN W. ALLEN

JOHN W. ALLEN, [21] JUN. AM. SOC. C. E.[21a]—An interesting comparison of three approximate methods of solving the problem of skew plates is presented in this paper. For the average engineer the difference-equation solution is the most direct of the three. The physical meaning of the difference equations can be understood readily and applied to various structures. This method often provides a powerful tool when exact solutions are not available.

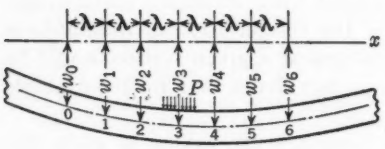

FIG. 7.—DEFLECTED BEAM

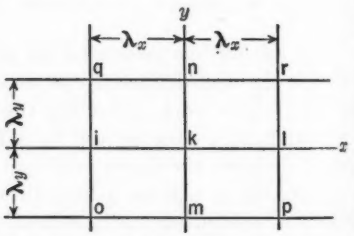

FIG. 8.—NETWORK OF POINTS IN A SLAB

If the structure under consideration were to deflect in only one direction such as the beam in Fig. 7, these equations can be set up in simplified form. The basic equations can then be transposed easily to a structure that is deflecting in two directions.

The difference equations are merely approximate expressions for the slope, and the rate of change of slope, of a curve such as a deflected surface or a moment curve. The theorem is basic, in ordinary structural analysis, that:

(I) The moment in a structure is proportional to the rate of change of slope of the deflected structure; and

(II) The load applied to a structure at a point is proportional to the rate of change of the slope of the moment curve.

NOTE.—This paper by F. L. Ehasz was published in June, 1945, *Proceedings*.

[21] Engr., Chicago Bridge & Iron Co., Chicago, Ill.

[21a] Received by the Secretary July 17, 1945.

The corresponding difference equations for the foregoing theorems can be written as follows: The rate of change of slope of a structure, at point 3, Fig. 7, is proportional to—

$$\frac{\text{(Slope midway between points 2 and 3)} - \text{(slope midway between points 3 and 4)}}{\text{Distance between the midpoints}}.$$

In other words,

$$\left(\frac{d^2w}{dx^2}\right)_3 \cong \frac{1}{\lambda}\left(\frac{w_2 - w_3}{\lambda} - \frac{w_3 - w_4}{\lambda}\right) \cong \frac{w_2 - 2\,w_3 + w_4}{\lambda^2} \quad \ldots\ldots (57)$$

but $\frac{d^2w}{dx^2} = -\frac{M}{EI}$; and therefore

$$\left(\frac{M}{EI}\right)_3 \cong -\frac{w_2 - 2\,w_3 + w_4}{\lambda^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

These same equations can be written similarly for the second theorem. For example, let P = the load on the structure at the point under consideration. Then $P = \frac{d^2M}{dx^2}$; and:

$$(P)_3 \cong \frac{M_2 - 2\,M_3 + M_4}{\lambda^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

By combining Eqs. 58 and 59 load P can be expressed in terms of the deflection w:

$$\left(\frac{P}{EI}\right)_3 \cong -\frac{w_1 - 4\,w_2 + 6\,w_3 - 4\,w_4 + w_5}{\lambda^4} \quad \ldots\ldots\ldots\ldots (60)$$

From the foregoing it can be seen that difference equations involve finite distances to express approximately that which differential equation expresses in minute distances. With difference equations the expressions are solved simultaneously for a few points instead of a complete solution for an infinite number of points.

It is also interesting to note the geometrical meaning of the twisting moment in the slab. Referring to Fig. 8, the following expression for the twisting moment can be obtained. Since the twisting moment is proportional to the rate of change of slope in the x-direction with respect to the y-direction, it can be expressed approximately for point k as follows:

$$\frac{\text{(Slope at point } n \text{ in the } x\text{-direction)} - \text{(slope at point } m \text{ in the } x\text{-direction)}}{\text{Distance between the points } n \text{ and } m \text{ in the } y\text{-direction}}.$$

In other words,

$$-\left[\frac{M_{xy}}{(1-\mu)N}\right]_k \cong \frac{1}{2\,\lambda_y}\left(\frac{w_q - w_r}{2\,\lambda_x} - \frac{w_o - w_p}{2\,\lambda_x}\right) \cong \frac{w_q + w_p - w_r - w_o}{4\,\lambda_x\,\lambda_y} \quad (61)$$

The foregoing demonstrates that difference equations supply a direct, even though approximate, method of computing quantities that are important in the

design of a structure. An equation must be written for each point on the network selected, and solved simultaneously.

Application of these equations and tests of models of structures[22] proves that the conclusions concerning the number of points on a network required to obtain reasonable accuracy are substantially correct.

[22] "Analysis of Skew Slabs with Curbs," by John W. Allen, thesis presented to the Univ. of Illinois, Urbana, Ill., in 1941, in partial fulfilment of the requirements for the degree of Master of Science in Theoretical and Applied Mechanics.

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DISCUSSIONS

CORRELATING FLOOD CONTROL AND WATER SUPPLY, LOS ANGELES COASTAL PLAIN

Discussion

BY A. L. SONDEREGGER

A. L. SONDEREGGER,[2] M. AM. SOC. C. E.[2a]—Clear statements of the principal methods applied in the utilization of flood waste by spreading and of the correlation of flood control and water conservation, as practiced by the Los Angeles County Flood Control District, are presented in this paper. Conservation by spreading is supplementary to, and an improvement on, the natural process of seepage from stream beds on the débris cone and also supplementary to the natural cyclic regulation of water supplies by underground storage which is peculiar to Southern California. It is this natural regulation which has made possible the development of this area and which has carried its population with immunity through a period of predominantly dry years which extended from 1922 to the latter part of the 1930's.

Referring first to the quality of conservation water, Table 3 shows what was to be expected—namely, that the storm runoff from the granitic watersheds of the Sierra Madre is stable, organically pure, and potable from the standpoint of total dissolved solids. The total hardness of 80 of Southern California storm water appears to be high, but compares favorably with the total hardness of 25 of the Mono Basin snow water and of aqueduct water of 96, which in part is fed from snow fields.

Cols. 2 and 3, Table 3, indicate that the comparatively short-time contact of the "after storm runoff" with the disintegrated soil cover of the watershed materially increases the percentage of dissolved solids and the hardness. This tends to prove that disintegration and decomposition of the granitic formation make some of the constituent components available for solution. This conclusion is further borne out by the fact that storage in ground-water basins and the long-period contact with alluvial deposits materially increase the dissolved solids. This is apparent from a comparison of the analysis of mountain water "during storm" (Col. 2, Table 3) and valley water "after storm" (Col. 5,

NOTE.—This paper by Finley B. Laverty was published in June, 1945, *Proceedings*.

[2] Cons. Engr., Los Angeles, Calif.

[2a] Received by the Secretary July 26, 1945.

Table 3), the latter being a mixture of "after storm" mountain runoff and rising ground water. The proportion of sulfur hydroxide increased from 28 ppm to 93 ppm, chloride from 24 ppm to 36 ppm, and total hardness from 80 ppm to 177 ppm. The relatively harmless increase in dissolved solids by underground storage is compensated by the preservation of the purity and potability of the water for indefinite periods.

Conditions, more severe, result from reservoir storage in the mountain watershed. Under natural conditions, the periodic scour of canyons by floods, small and great, removes decayed vegetable matter, ashes, and other impurities. With the interception of all débris, fine and coarse, in storage reservoirs, or behind débris barriers, this flushing process remains incomplete, with the result that silt and ashes accumulate above dams, decaying débris is piled up in stagnant pools, and a growth of a multiplicity of organic life may be induced, including Crenothrix. Nevertheless, the more important fact remains that water supplies are conserved by impounding, and if they are of inferior quality modern methods of purification are able to cope with them.

The volume of water that can be salvaged in wet years by spreading is substantial and is directly dependent upon available storage and the methods of operating flood-control reservoirs. The most complete plans for flood control and conservation are those for the San Gabriel River prepared jointly by the Los Angeles County Flood Control District, and the U. S. Engineer Department. Available storage as of 1945 is as follows:

Dam	Storage in acre-ft
San Gabriel No. 2	12,000
San Gabriel No. 1	47,000
Morris	36,000
Santa Fe	30,000
Whittier Narrows	....

Assuming the proportion of the total storage at Morris Dam available for flood control as 18,000 acre-ft, and the proportion of the combined total storage at Santa Fe Dam and Whittier Narrows Dam as 10,000 acre-ft, the total storage available for purposes of conservation will be 8,500 acre-ft. Under these conditions the Flood Control District estimates the net seasonal salvage available for the valley above the Narrows at 3,000 acre-ft and for the coastal plain at 12,700 acre-ft. The total is 15,700 acre-ft, which is 11.5% of the mean seasonal runoff of 136,000 acre-ft at the mouth of the canyon and 18½% of the conservation storage.

In 1924 an estimate of possible salvage of flood waste was made for a report on the water supply of the metropolitan area of Los Angeles by a board of engineers composed of the late J. B. Lippincott, Hon. M. Am. Soc. C. E., Louis C. Hill, Past-President, Am. Soc. C. E., and the writer. Under the assumption of an available storage of 200,000 acre-ft then planned at San Gabriel site No. 1, the estimated salvage for San Gabriel River watershed was 34,000 acre-ft per acre or 17% of the conservation storage, and this quantity was credited in the report to the ultimate local water supply available for the Los Angeles metropolitan area.

Mr. Laverty refers to the first spreading development in Los Angeles County, on San Antonio Creek cone above Pomona in 1906. An interesting story is connected with the initiation of spreading in that area. In the 1890's, there was a beeman by the name of Arenz, of whom it was said that he truly had "a bee in his bonnet." Arenz noticed that along the wet washes in the upper San Antonio Creek cone there was a more vigorous growth of brush and a more luxurious bloom than along the dry channels. More bloom meant more food for the bees, and more food for the bees meant more honey in the hives; so in order to promote the growth of brush over wider areas he conceived the ingenious idea of diverting and spreading the stream artificially. This practice continued over some years and as a matter of fact ripened into a right of appropriation. After the turn of the century, a great development of citrus began on the cones in that vicinity, after it had been proved that the débris deposits were an excellent medium for the propagation of citrus, particularly lemons. The ranchers, in need of more water supplies, realized the advantage of recharging the underflow by spreading the flood waters and a general movement for the appropriation of flood waste began. Some bright men realized that the beeman had acquired a priority to a substantial portion of the creek flow of the early spring months and they promptly bought him out. They continued the diversions, not for the propagation of brush however, but for ground-water recharge and in a subsequent action in the courts in 1912 successfully maintained their position and priority by proving that the spread water percolated to their wells and fed their ciénagas. The late F. E. Trask, M. Am. Soc. C. E., was the first engineer to practice water spreading in Southern California in the nineties.[3]

The question naturally arises as to the legal status of the spread water. A water right is established generally by diversion for beneficial use. The Los Angeles County Flood Control District is charged by Legislative Act with the conservation of flood water; but the District is not a water user and hence not an appropriator. After the water is diverted and spread, it changes its status from that of surface flow to that of percolating ground water and is then available for appropriation to overlying lands or for export. In most instances where conservation is practiced in this area, the diversions are made by incorporated conservation and protective associations who are not water users, but operate for their volunteer membership. Spread water mingling with natural seepage loses its identity, although not necessarily its status as appropriated water, provided the appropriator can trace his spread water to the point of abstraction. In the case of these associations, the member water users are not the original appropriators. Moreover, their wells may be many miles from the spreading grounds and widely distributed over the ground-water field. In closed basins, it is believed that the appropriator and impounder of floodwater may legally defend his right.

In the case of Pasadena vs. Alhambra,[4] relative to the adjudication of rights to the water supply of the Raymond ground-water basin in the Pasadena area,

[3] "Water Conservation in Southern California," by F. E. Trask, *The Rural Californian*, 1903.

[4] No. Pasadena C-1323 in the Superior Court of Los Angeles County, California, decided December 23, 1944.

rights were established on the basis of pumping abstractions for beneficial use; no credit being awarded for the spreading of local supplies. As to possible rights to imported water, if spread in the basin, the situation is left open as indicated by the following quotation[5] from said judgment:

> "That nothing in this judgment contained shall be considered as a determination of the right of any party to water hereafter imported by it into the Raymond Basin Area and spread therein, with the intent to recapture the same."

After the destructive floods of 1914 and 1916, Los Angeles County undertook a program of flood control by the construction of reservoirs with the dual intent of regulating peak flow and conserving floodwaters. As a matter of fact, bond elections were carried on the issue of water conservation. A majority of the population had moved in after 1916 and were not familiar with local flood conditions; hence they did not grasp the necessity for flood control. However, they did realize the effects of dry years and the need for conserving flood waste.

During the recent protracted period of drought, public opinion and pressure were sufficiently strong to give conservation precedence over flood control and reservoirs were operated accordingly. Since the flood of 1938 a more cautious attitude in operating detention reservoirs has been adopted.

It is concluded, in the disposal of flood waste, that preference be given to: (1) Flood control, (2) conservation, and (3) quality of water—in the order of their economic importance.

The writer's opinion as to the relative importance of flood control and conservation may be expressed by paraphrasing one of the famous sayings of the late William Mulholland, M. Am. Soc. C. E., who, when campaigning in 1906 for a bond issue of $23,000,000 to finance the Los Angeles (Owens Valley) Aqueduct, told his audience: "If you don't get the water, you won't need it." If Southern California does not get flood control, it will not need conservation.

[5] No. Pasadena C-1323 in the Superior Court of Los Angeles County, California, decided December 23, 1944, paragraph XVIII, p. 23.

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DISCUSSIONS

MATHEMATICAL EXAMINATION OF THE HIGHWAY TRANSITION SPIRAL

Discussion

BY JOSEPH BARNETT, RALPH W. STEWART, DONALD THOMPSON, T. F. HICKERSON, ALLEN H. BROWNFIELD, RENÉ S. PULIDO Y MORALES, AND C. C. WILEY

JOSEPH BARNETT,[4] M. AM. SOC. C. E.[4a]—In several respects this paper is valuable, and not the least of these is its importance in calling the attention of highway engineers to a neglected phase of geometric highway design. Despite the surprisingly large bibliography, for which engineers are indebted to the authors, the use of transition approaches to curves on highways has not been generally accepted even though greater safety, smoother riding, and a more graceful alinement are attained without extra cost. Only about half the state highway departments now use transitions in highway alinements.

Among the reasons for this are reluctance to change existing practices and to revise standards; a belief that a vehicle using the highway is not confined to a track and is able to effect the change from a straight to a circular motion smoothly; a belief that widening at a sharp curve together with the gradual attainment of this widening accomplishes a satisfactory transition; and a belief that transitions involve tedious calculations both in the office and field, resulting in increased costs of engineering. There has also been considerable confusion as to what factors should determine the minimum length of a transition.

Some of these reasons were valid when road speeds were slower than at present but they do not apply now. At moderate speeds the average driver can traverse a satisfactory short transition path within the limits of a vehicle lane, but with increased speeds longer transitions are required. These can be traversed only by hazardous crowding or occupation of adjoining lanes. The increase in the speeds at which vehicles are operated, the desire for increased speed with safety, and the mounting accident toll make it imperative that highways be constructed in such manner that a driver traveling at the safe

NOTE.—This paper by John Oran Eichler and Howard W. Eves was published in May, 1945, *Proceedings*.

[4] Chf., Urban Road Div., U. S. Public Roads Administration, Federal Works Agency, Washington, D. C.

[4a] Received by the Secretary June 11, 1945.

speed for which the highway is designed will not only find it possible to confine his vehicle to the occupied lane but will be encouraged to do so.

A discussion of the highway transition spiral might be divided into two general sections—theoretical and practical. Too much already has been written regarding theory. Several forms of curve have been advocated. The form used by the authors is that in which the radius varies inversely as the distance from the beginning of the transition curve. It is the same form of transition used by the late Arthur N. Talbot, Past-President and Hon. M. Am. Soc. C. E. (29),[45] except that Professor Talbot developed a set of tables around the symbol a which represents the rate of change of degree of curve per 100 ft of length, and which equals the degree of curve at a point 100 ft from the "TS," the common point of tangent and spiral. T. F. Hickerson, M. Am. Soc. C. E. (13), used the same form of transition as the authors and gave the mathematical derivation for the coordinates x and y, but extended the mathematical derivation no further, since almost all values required for tables can be computed from x and y. W. H. Searles (25) developed a transition effected by a series of compound circular curves. In England, F. G. Royal-Dawson (23) advocated the use of the lemniscate, a form of curve which permits a certain type of field simplification. Several other forms are available. As regards riding qualities, safe speeds, or rates of change of superelevation, no one form of transition has any particular advantage over other forms.

There have been differences of opinion regarding the length of transition which should be used in highway design. Suggested lengths have been developed by theory but unfortunately there have been differences regarding the basis on which theoretical lengths should be developed. In developing lengths of transition for railways, W. H. Shortt used an average rate at which the vehicle on the transition approaches the final constant centripetal acceleration on the circular curve (27), but the rate to be used on highways is largely a matter of opinion. In a similar approach the time to turn the steering wheel to attain a certain degree of curvilinear motion at a certain speed has been advocated as a basis for length determination. Rate of runoff (that is, the change of superelevation in a unit of length) has many advocates, but there are differences of opinion regarding both the rate of runoff and the rate of superelevation. In all these determinations the speed of the vehicle is assumed and that is the basic difference of approach, particularly between designers in England and in the United States.

In England most designers assume that the geometric features of each curve, the radius, superelevation, and length of transition, determine the speed of the vehicle, an idea which has many advocates in the United States. Under this assumption a driver will change his speed at each curve. Although scientifically controlled observations are needed for a proper determination, most engineers have observed that once the average driver escapes the influences of urban conditions that tend to force a wide range of speeds, he is inclined to settle down to a uniform speed which, in his mind, is as great as the traffic and the physical characteristics of the highway permit. The speed is seldom the

[45] Numerals in parentheses, thus: (29), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

maximum speed of the vehicle since almost all passenger vehicles and some buses and trucks can attain speeds greater than that considered safe on existing highways and greater than that for which the public and highway engineers deem it economically feasible to construct most highways at the present time. For purposes of design it is assumed that traffic is not dense enough to slow down a vehicle and that the steady speed adopted by the driver depends largely upon the physical characteristics of the highway.

The personal factor cannot be ignored. Some drivers choose higher speeds than others. Some passengers feel comfortable at higher speeds than others. Those physical characteristics of the highway that tend to make a fast driver slow down also tend to make a slow driver reduce speed; but an approximately uniform speed, rather than a variable speed changed at each curve, is generally the aim of all drivers.

The difference in thinking regarding speed to be used in design has led to still another difference—one which relates to the use of a curve transitional throughout. The idea that each curve is traversed at its maximum permissible speed leads to use of curves transitional throughout for the smaller values of the intersection angle, Δ, and has been advocated chiefly in England. Highway designers in the United States incline to the use of two transitions and a central circular curve each taking up about a third of the central angle.

These differences can be reconciled only by the application of the results of tests of speeds on highways generally, tests of speed in relation to curvature and superelevation, and tests of driving comfort around curves. Results of tests on speed and comfort have been reported by D. W. Loutzenheiser (36), O. K. Normann (37)(38), and A. Taragin (39). The tests by John J. Leeming and A. N. Black (40) constitute one of the few attempts to measure comfort and driving practice around curves and, although they leave much to be desired to make them completely scientific and conclusive, they point the way toward conclusions which the writer feels are significant and which he has assumed for many years as the result of sheer logic. In their conclusions, Messrs. Leeming and Black state:

> "* * * it is tentatively suggested that curves should be designed to have about two-thirds of their length transitional, which would mean that the transition angle would be about one-quarter of the deviation angle. Curves designed according to this rule would then conform to what appears to be the average practice of drivers. As far as the information goes at present, it does not appear that any better compromise is available. As, however, the length of transition is not very important, it will be amply good enough for all practical purposes to make the transition angle as near as convenient to one-quarter of the deviation angle. In general, transitions should be short rather than long, and the main object should be simplicity and ease of layout and construction."

It is evident that precision in the form or length of transition curve is not necessary. The differences of opinion regarding the factors that determine length and type (particularly the factor of speed and its relation to curvature and superelevation) prove this. Then the problem of encouraging the adoption of transition curves for highway alinements is not one of persuading highway

engineers to use a particular type of curve or particular relationship that results in a particular length, but in persuading engineers to use some kind of a transition curve. The mathematics presented by the authors should be simple to any one who has completed the average college course in calculus, or even advanced algebra; but it will "scare the shirt off" many surveyors on highway work. In addition the calculations must be made frequently in the field and supervisors are discouraged from adopting any procedure that requires a survey party to await the results of tedious calculations.

The control on most highway alinement problems, once the deviation angle Δ is measured, is either the tangent distance, T, or the external distance, E, from the point of intersection to the center of the curve, or both. If the common practice is followed of first designing and locating a circular curve and later adjusting the curve for inclusion of transition approaches, it results in odd values of the degree of curve and length of transition, and tedious calculations are necessary. If the uselessness of precision in length of transition is recognized, however, tables can be prepared which give the tangent distance T_s for curves with transitions, and the external distance E_s for curves with transitions, directly, and it becomes as easy with such tables to design and locate curves with transitions as to design and locate simple curves, provided the curves are designed initially as curves with transitions. Such tables, referred to by the authors (2), were prepared by the writer in 1938.

Several years of experience with these tables have revealed wherein they have proved their worth and where there are shortcomings. Where they have been tried on actual work and the designer or locator has jumped the hurdle of the explanatory mathematics which was kept to the minimum, no difficulty was encountered. In fact, the writer now advises that initiates omit the text and proceed to actual trial using the numerical examples to learn the simple operations. The choice of lengths of transition is limited to multiplies of 50 ft and the degrees of curvature to even degrees, in some cases half degrees, so that except for straight-line interpolation no calculations are necessary that cannot be made, for all practical purposes, mentally; and, for every combination of the length of spiral L_s and the degree of curve D_c additional tables give all salient features of the transition curve including deflection angles to points on the transition divided into ten equal lengths.

Incidentally, although mathematicians do not take kindly to the concept of the degree of curvature used by the writer, it is a mighty handy tool to the man who must make calculations quickly. The length of arc subtending a central angle to be known as the degree of curve should be an even 100 ft and not 100.007 ft which the authors apparently chose in order to make the value representing the relationship between radius and degree of curve come out even. The necessity for having this value even is obscure. However, the use of radius instead of degree of curve does not minimize the usefulness of the type of tables proposed by the writer. In their Spanish translation in 1941, M. San Miguel, E. F. Tagle, and A. J. L. Bolognesi used the radius of curve in recomputing the writer's tables (2) to the metric system.

The principal shortcomings revealed by use of the tables in "Transition Curves for Highways" (2) are two in number: (*a*) For the larger values of Δ

small values of D_c will seldom be used and corresponding values of T_s and E_s are unnecessary. It would be better to devote the limited space to giving a greater choice of the larger values of D_c; that is, values corresponding to degrees and half degrees instead of degrees only; and (*b*) for the very large values of Δ used in mountainous areas, and sometimes in turnouts, values for $L_s = 100$ ft and possibly values for sharper curvatures should be added.

RALPH W. STEWART,[5] M. AM. SOC. C. E.[5a]—As a mathematical examination of the transition spiral this paper is very thorough and complete. The bibliography, although unusually well prepared, omits what is probably the first presentation for practical use in America of the mathematics of the true transition spiral whose curvature uniformly increases from point to point. This was a paper (41) by the late Elliot Holbrook, M. Am. Soc. C. E., which appeared in 1880. The same mathematical procedure was used by A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E. (42), in a paper published in 1891, later to be included in Professor Talbot's book, "The Railway Transition Spiral" (29). In 1899, F. K. Vial used the law of constant increase in curvature of the transition spiral to devise a universal table of coefficients for deflections for laying out the curve. By the use of this table of coefficients (43) (29*a*), an instrument could be set at any chord point of any spiral which was divided into uniform chord lengths and the deflections to all other chord points could be computed by multiplying the spiral constant by the series of coefficients.

There were, in former years, many engineers who boycotted such mathematical procedures as the expansion of a function into an infinite series of terms by Maclaurin's theorem or otherwise, and who were annoyed by formulas such as Eqs. 10 and 11. This led to the use of the curve composed of a series of finite circular arcs whose properties could be computed by trigonometry.

The author states that he does not propose to enter into arguments favoring any one type of easement. The writer, however, wishes to call attention to the disadvantages (discovered through actual field experience) of the curve composed of a series of finite circular arcs; and also to show how to lay out a true spiral curve which, for all practical purposes, will coincide with a multi-arc curve.

The chief disadvantage of the multi-arc curve is that, with the transit set at the beginning of the first arc, which is called the beginning of the curve, accurate deflections are known only to the points where the arcs are compounded. If it is necessary to set points for the bents of a trestle bridge, a common procedure is first to set stakes at the compound points of the arcs and then move the transit from compound point to compound point to set the intermediate points at the bent locations. This and similar situations are not rare. It is sometimes necessary to set points that do not coincide with the compound points in order to investigate side clearances and to locate track crossings or other intersecting lines. The accumulation of all such incidents is quite appreciable. At one time the writer had to lay out the points of a multi-

[5] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

[5a] Received by the Secretary June 11, 1945.

compound-arc spiral on the floor beams of a high steel bridge which was under construction. Not knowing at that time how to run the equivalent true spiral it was necessary for the bridge construction crew to lay temporary plank platforms at the compound points of the curve.

The true spiral, which (for all practical purposes) will coincide with a multi-arc substitute curve, can be determined easily. It is only necessary to lay out one arc length of tangent behind the beginning of the first arc. The center point of this tangent and the center points of the consecutive arcs will then define a true spiral of ample accuracy for practical use which will have the same rate of increase in curvature per unit arc length as the multi-compound substitute curve.

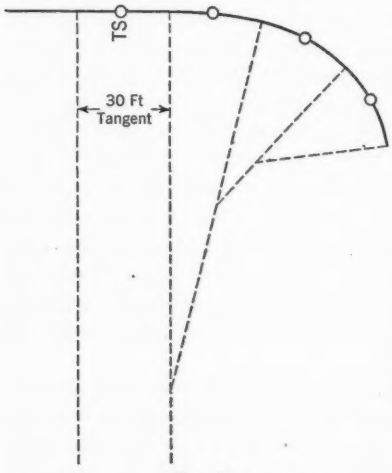

FIG. 6.

This is best illustrated by a numerical example. Let Fig. 6 represent a compound curve composed of arcs each 30 ft long. Let the first arc be a 0° 30′ curve, the second a 1° curve, etc. Note that this provides for an increase in the rate of curvature of 0° 30′ in 30 ft or 1′ per ft. The radial lines in the figure indicate the ends of the arcs. The small circles indicate the centers of the arcs, and also the center point, marked TS (change from tangent to spiral), of the arc length of tangent which precedes the curve. If the accumulated curvature from TS to the center of each arc is computed, it will be found to vary as the square of the distance along the center line which is the intrinsic equation of the true spiral. It will also be found that a true spiral having a rate of curvature increase of 1′ per ft and beginning at the TS point will pass through the center points of the arcs, subject to negligible errors depending on whether chord lengths or arc lengths are measured and other causes of minute differences.

Since ample tables are available for laying out spirals, it is not necessary to cope with equations that involve an infinite series of terms.

The following analogy is useful in general engineering practice: A transition spiral represents uniform increase in rate of curvature. Uniformly accelerated motion represents uniform increase in velocity. This makes the total curvature in a spiral for any distance x along the curve equal to $0.5\ a\ x^2$, in which a is the rate of increase in curvature. This is analogous to the formula $0.5\ g\ t^2$ which represents the distance that a body falls (starting from rest) in any elapsed time t under the rate of acceleration g. Also, the elastic curve of a cantilever beam of uniform section caused by a concentrated load at its free end is a transition spiral as long as the deflection is small, because the bending moment, and consequently the elastic curvature, increase uniformly with the distance from the load.

DONALD THOMPSON,[6] ESQ.[6a]—The review of the mathematical theory of the highway transition spiral, as presented in this paper, should prove interesting and instructive to engineers designing high-speed highways. Recent publications on transition curves, notably the work of Joseph Barnett, M. Am. Soc. C. E., cited by the authors (2), have avoided an exposition of the mathematical theory on which the true transition curve is based. Thomas F. Hickerson, M. Am. Soc. C. E. (13), gives the derivation of most of the spiral formulas but omits the mathematical theory for the formula for the deflection angle.

Earlier publications on the true transition curve, however, gave the derivation of all the fundamental equations developed by Professors Eichler and Eves. In 1893 the late C. L. Crandall, M. Am. Soc. C. E., (44) developed all the formulas for the true transition curve. Professor Crandall credits the late Elliot Holbrook, M. Am. Soc. C. E., with first describing the true transition curve in which curvature increases directly with the distance (41), and developing the formulas and practice for small angles.

It is true, of course, that knowledge of the mathematical theory is not essential either for designing or for staking transition curves. With either Mr. Barnett's or Professor Hickerson's book, the use of transition curves will involve scarcely any more work than the use of circular curves. The authors' method of calculating the elements of a particular spiral by substituting in the fundamental formulas, although interesting from a mathematical standpoint, is not practicable. The spiral elements can be taken directly from tables in either of the afore-mentioned books (2) (13) in a fraction of the time consumed in substituting appropriate values in the formulas of this paper.

The derivation of the formula for the deflection angle α in Section 5 is not complete. Eq. 7 was given as $\tan \alpha = \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{\phi^5}{5{,}597}$, in which ϕ is in radians. To derive a formula for α (not $\tan \alpha$) the deflection angle must be expanded in terms of its tangent. Thus,

$$\alpha = \tan \alpha - \frac{1}{3} \tan^3 \alpha + \frac{1}{5} \tan^5 \alpha \ldots\ldots\ldots\ldots\ldots (34a)$$

Substituting Eq. 7 in Eq. 34*a*, dropping higher powers than the third, and converting to degrees, the correct formula for the deflection angle is

$$\alpha^\circ = \frac{\phi^\circ}{3} - 0.000000859\ (\phi^\circ)^3 \ldots\ldots\ldots\ldots\ldots\ldots (34b)$$

The formula for the correction C that must be subtracted from one third of any given value of ϕ in order to obtain the corresponding value of α is, therefore,

$$C = 0.000000859\ (\phi^\circ)^3 \ldots\ldots\ldots\ldots\ldots\ldots (35a)$$

In calculating the deflection angles for a particular spiral it is necessary to express the deflection angle α in terms of the length s along the curve to the point for which the deflection is desired, the total length S of the spiral, and the

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[6a] Received by the Secretary June 18, 1945.

total spiral angle Φ. In terms of these variables

$$\alpha = \frac{\Phi^\circ}{3}\left(\frac{s}{S}\right)^2 - 0.000000859\,(\Phi^\circ)^3\left(\frac{s}{S}\right)^6 \quad \ldots\ldots\ldots\ldots (35b)$$

The formula for the centrifugal ratio, Eq. 15, is given as $\frac{V^2}{g\,R} = \frac{e+f}{1-ef}$ in which e is the superelevation in feet per foot, f is the coefficient of friction, R is the radius in feet, g is acceleration of gravity, and V is the velocity in feet, per second. This is the exact formula for superelevation with friction considered. It would be useful in designing a racing track where large values of e might be necessary, but it is never used in highway design. Since the maximum values of e and f permitted in the design of highways are quite small, $1 - ef$ will be very nearly equal to 1 and the formula for the centrifugal ratio can be simplified to

$$\frac{V^2}{g\,R} = e + f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36a)$$

Converting V to miles per hour, Eq. 36*a* becomes

$$\frac{0.067\,V^2}{R} = e + f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36b)$$

which is the approximate formula for the centrifugal ratio and the one always used in highway design. Since the approximate formula will give slightly lower velocities than the exact, it is on the side of safety.

A value of $f = 0.14$ for the unbalanced centrifugal ratio (which is mentioned by the authors in Section 7 as being the accepted standard) seems too great for comfort at high speeds. Ralph A. Moyer, Assoc. M. Am. Soc. C. E. (45), has recommended that the unbalanced centrifugal ratio f should not exceed 0.10 at 60 miles per hr. Charles M. Noble, M. Am. Soc. C. E. (46), after exhaustive high-speed tests on the Pennsylvania Turnpike, recommended that, for velocities of 70 miles per hr, f should not exceed 0.10. In designing parkways for the Bureau of Engineering of the City of Los Angeles, Calif., the writer uses a value for f which varies with the velocity. The variation is considered to be linear and, expressed as a formula, is

$$f = 0.16 - 0.001\,V \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

in which V is in miles per hour. At 60 miles per hr, f would be 0.10. Using a maximum value of 0.10 for e, the maximum value of the centrifugal ratio would be 0.20. For the maximum superelevation of 0.10 the minimum radius will be $R = \frac{0.067 \times 60^2}{0.20} = 1{,}200$ ft. For curves larger than the minimum the superelevation should vary inversely as the radius so that the general formula for superelevation for a design speed of 60 miles per hr is

$$e = \frac{120}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

in which e is in feet per foot, and R is in feet. For the length of the transition

spiral and the superelevation transition the writer is using a value of 1.5775 ft per sec^3 for γ instead of the commonly accepted 2 ft per sec^3 which results in a value for the length of the transition of

$$S = \frac{2\,V^3}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39a)$$

in which V is in miles per hour and S and R are in feet. For a design speed of 60 miles per hour, Eq. 39a can be expressed as

$$S = \frac{432{,}000}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39b)$$

This length of transition will limit the difference in grade throughout the superelevation transition between the center line of a 36-ft roadway and the outside edge of a 36-ft roadway to 0.5% or 1 in 200 provided the superelevation is attained by rotating about the center line of the roadway.

It is the present practice in the City of Los Angeles to first project a circular alinement making proper allowances for external distances and lengths of tangents between reversing curves and then to insert the transition curves. With this method it becomes a very simple matter to determine whether a curve will be transitional throughout or have a remaining circular arc. If any circular arc on the projected alinement has a length equal to 432,000/R, the transition curves when inserted will take up all the circular arc and the resulting curve will be transitional throughout. If a circular curve on the original alinement has a length greater than 432,000/R, then there will be some circular arc left after the transitions have been inserted. For the sake of appearance, the minimum length of any circular curve has been set at 500 ft. Since the maximum length of transition would be 432,000/1,200 = 360 ft, occurring on the minimum radius of 1,200 ft, all curves will have a portion of a circular arc remaining and will not be transitional throughout.

In their conclusion the authors state that they hope their paper will serve as a clear and concise introduction to the fundamental principles of the true transition curve. The paper has accomplished its purpose and should stimulate interest in, and promote, the wide acceptance of the true transition curve by highway engineers.

T. F. HICKERSON,[7] M. AM. SOC. C. E.[7a]—In their mathematical examination of the highway transition spiral the authors have cited some, but not by any means all, of the contributions that have been made in adapting this curve to the layout of highways. For example, the writer has prepared a table containing the deflections from a tangent to any point on a spiral (13a). This table meets the objection mentioned by the authors, in their description of two methods of staking the spiral (Section 12), that "the points fixed do not coincide with the even chain lengths along the curve * * *." In connection with the curve that is transitional throughout, the writer prepared a table

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[7a] Received by the Secretary July 27, 1945.

(13*b*) giving "Tangents and Externals to the Unit Double-Spiral Curve of Length 100 Ft."

Knowing the intersection angle Δ and the actual length of this curve which is transitional throughout, the tangents and externals can be written at once. Thus, if $\Delta = 40°$ and the length of each spiral is 600 ft (total length of curve 1,200 ft), then $T_s = 102.987 \times 6 = 617.92$ ft and $E_s = 12.275 \times 6 = 73.65$ ft. These tabular values of T_s and E_s for the unit curve were computed by the writer from the formulas:

$$T_s = T + k + p \tan 0.5\,\Delta \ldots\ldots (40a)$$

and

$$E_s = E + p \sec 0.5\,\Delta \ldots\ldots (40b)$$

in which T and E are the tangents and external distances for the osculating circular curve;

$$k = x_c - R_c \sin \phi_c \ldots\ldots (41a)$$

and

$$p = y_c - R_c \operatorname{vers} \phi_c \ldots\ldots (41b)$$

In Eqs. 41 the subscripts c refer to point P (Fig. 1) at the end of the spiral where it coincides with the circle.

Eqs. 40 are more simple than Eqs. 21 presented by the authors inasmuch as the data required for their solution are already known to a high degree of accuracy (13*c*). The tabular values of T_s and E_s have been checked by the writer by means of the following independent pairs of formulas:

$$T_s = x_c + y_c \tan \phi \ldots\ldots (42a)$$

$$T_s = c\,(\cos \alpha + \sin \alpha \tan \phi) \ldots\ldots (42b)$$

$$E_s = y_c \sec \phi \ldots\ldots (43a)$$

$$E_s = c \sin \alpha \sec \phi \ldots\ldots (43b)$$

The values of x and y for any point of the spiral are:

$$x = s\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \frac{\phi^6}{9{,}360} + \cdots\right) \ldots\ldots (44a)$$

and

$$y = s\left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1{,}320} - \frac{\phi^7}{75{,}600}\right) \ldots\ldots (44b)$$

in which ϕ is measured in radians.

Expressing ϕ in degrees, Eqs. 44 become:

$$x = s\,[1 - 0.304617\,(10)^{-4}\,\phi^2 + 0.429592\,(10)^{-9}\,\phi^4 - 0.301987\,(10)^{-14}\,\phi^6 + \cdots] \ldots\ldots (45a)$$

and

$$y = s\,[0.581776\,(10)^{-2}\,\phi - 0.126585\,(10)^{-6}\,\phi^3 + 0.122691\,(10)^{-11}\,\phi^5 - 0.65256\,(10)^{-17}\,\phi^7 + \cdots] \ldots\ldots (45b)$$

Calculating machines and eight-place tables of logarithms and trigonometric functions are needed to compute the tabular values correct to the sixth

significant figure. Such accuracy would establish the parts of a spiral to the nearest hundredth of a foot even when the length of the spiral is 1,000 ft. Now that these tables have been compiled and checked independently by different parties it is to be hoped that engineers will use them in good faith just as they do tables of logarithms.

The authors brought up a good point in stating that there are any number of permissible spirals suitable for a given velocity V and intersection angle Δ. In Eq. 16*b*, if $\gamma = 2 \frac{\text{ft}}{\text{sec}^3}$ (a safe limiting value) and if $V \frac{\text{ft}}{\text{sec}}$ is expressed as $v \frac{\text{miles}}{\text{hr}}$:

$$S = \frac{1.58\, v^3}{R} \qquad (46)$$

Eq. 46 is often misinterpreted. It gives the minimum permissible length of transition curve that may be used without exceeding the maximum allowable rate of change of acceleration while driving a car from a straight path to the full curvilinear path.

Assume that $v = 60$ miles per hr; then if $R = 2{,}000$ ft, $S = 170.6$ ft; and if $R = 4{,}000$ ft, $S = 85.3$ ft. These values of S are strictly according to the arbitrary formula, but if a much longer transition is feasible the change from a straight to a curved path would be all the easier and the general appearance of the curve all the more improved.

ALLEN H. BROWNFIELD,[8] ASSOC. M. AM. SOC. C. E.[8a]—This paper and its extensive bibliography should prove quite useful in highway design for establishing departmental standards involving transition curves. In addition to the authorities cited previously the following should be included: R. C. Archibald (47), D. E. Hughes (48) (49), the late William Cain, M. Am. Soc. C. E. (50), the late C. L. Crandall, M. Am. Soc. C. E. (44), the late S. W. Robinson, M. Am. Soc. C. E. (51), who cites the work of the late Elliot Holbrook, M. Am. Soc. C. E. (41), Southern Pacific Railroad Company (52), J. C. Nagle (53), who cites the work of Mr. Crandall (44), the late W. B. Lee, M. Am. Soc. C. E. (54), John H. Fletcher (55), the late N. B. Kellogg, M. Am. Soc. C. E. (56), A. L. Higgins (57), H. Criswell (58), and William T. Pryor, Assoc. M. Am. Soc. C. E. (59). Each of these articles deals directly with Euler's spiral (sometimes referred to as the American spiral) and will help to trace its development in the United States.

A complete treatment of the Euler spiral will involve three steps: (*a*) A discussion of the background, as has been done so ably by the authors; (*b*) analyses by tables or graphs showing the authors' recommendations regarding length of curve and superelevation (also a more extensive list of useful formulas correlating all parts of the spiral to be used by highway planners); and (*c*) solutions of numerical examples for field and office use in staking this curve.

There seems to be considerable hesitancy on the part of some highway departments to adopt the Euler spiral. Maybe this is due to the apparent com-

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[8a] Received by the Secretary August 21, 1945.

plexity of the formulas and difficulty in finding the proper tables. Several of the references have quite extensive tables and one of these is a seven-place table of natural functions at 3-min intervals.

From the following it should be clear that the spiral, with tables, is nearly as simple as a circular curve, and more simple than a compounded curve. Evidently the circle would be difficult without extensive tables of natural and logarithmic functions.

Since most of the spiral tables are developed in terms of s and ϕ, it is convenient to compare them with a circle using the x-axis along a tangent, as follows:

Dimension	Circle (Fig. 7(*a*))	Euler's spiral (Fig. 7(*b*))
s	$r\,\phi$	$2\,R\,\phi$
y	$s\,\dfrac{\text{vers}\,\phi}{\phi}$	$s\,\text{vers}_s\,\phi$
x	$s\,\dfrac{\sin\phi}{\phi}$	$s\,\sin_s\,\phi$
c	$\dfrac{2\,s}{\phi}\sin\dfrac{\phi}{2}$	$s\,f(\phi)$
t (and t_1)	$\dfrac{s}{\phi}\tan\dfrac{\phi}{2}$	$s\,\tan_s\,\phi$
α	$\dfrac{\phi}{2}$	$\dfrac{\phi}{3}$ − correction

The terms $\sin_s$, vers_s, and $\tan_s$ are introduced to show the parallelism and are defined as "sine spiral," "versine spiral," and "tangent spiral," respectively. They are similar to the circular functions of $\dfrac{\sin\phi}{\phi}$, etc.

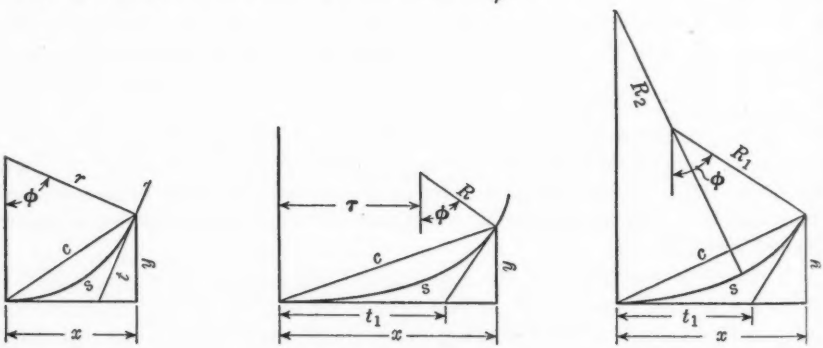

FIG. 7

From an examination of Figs. 7(*b*) and 7(*c*) it is evident that the tables for Euler's spiral are faster to use than those for a compound circle.

In the following discussion the writer has not included the derivations as they may detract from, rather than enhance, the use of spirals.

Apparently the authors followed the same procedure as F. G. Royal-Dawson (23) in deriving Eq. 7. If the division is made as indicated in the foregoing a formula is evolved which is nearly identical to Eq. 7:

$$\tan \alpha = \frac{y}{x} = \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{26\,\phi^5}{155{,}925} - \frac{17\,\phi^7}{3{,}378{,}375} - \cdots \quad (47)$$

or

$$\alpha = \frac{\phi}{3} - c = \frac{\phi}{3} - \frac{8\,\phi^3}{2{,}835} - \frac{32\,\phi^5}{467{,}775} - \frac{128\,\phi^7}{83{,}284{,}288} - \cdots \quad (48)$$

The last term of Eq. 48 is nearly correct; thus:

$$\phi = 3\,\alpha + \frac{8}{35}\,\alpha^3 + \frac{1{,}376}{13{,}475}\,\alpha^5 + \frac{207{,}488}{3{,}678{,}675}\,\alpha^7 + \cdots \quad (49)$$

and

$$c = s\left(1 - \frac{2\,\phi^2}{45} + \frac{2\,\phi^4}{2{,}835} - \frac{316\,\phi^6}{127{,}702{,}575} + \cdots\right) \quad (50)$$

Solution of Eqs. 7 to 13 will seldom be needed in practice since all values necessary for their solution have been tabulated by several authors.

The writer has found the following equations useful on several occasions (one or two terms will usually suffice). Let σ equal the offset AE in Fig. 3; then:

$$n = \frac{6\,\sigma}{r}\,\phi = \sqrt{n + \frac{n^2}{28} + \frac{29\,n^3}{16{,}170} + \frac{1{,}391\,n^4}{13{,}582{,}800} + \cdots} \quad (51)$$

The area under the curve between s, x, and y equals

$$A = s^2\left(\frac{\phi}{12} - \frac{\phi^3}{42} + \frac{23\,\phi^5}{10{,}395} - \frac{2\,\phi^7}{17{,}325} + \cdots\right) \quad (52)$$

In Fig. 2(*b*),

$$T = s\left(1 + \frac{7\,\phi^2}{30} + \frac{139\,\phi^4}{1{,}512} + \frac{8{,}927\,\phi^6}{240{,}240} + \cdots\right) \quad (53)$$

As compared with Eq. 22*b*,

$$\tau = s\left(\tfrac{1}{2} - \frac{\phi^2}{60} + \frac{\phi^4}{2{,}160} - \cdots\right) \quad (54a)$$

or

$$\tau \approx \frac{2\,s + x}{6} \quad (54b)$$

Similarly (see Eq. 22*a*),

$$\sigma = \frac{s}{2}\left(\frac{\phi}{3(2!)} - \frac{\phi^3}{7(4!)} + \frac{\phi^5}{11(6!)} - \frac{\phi^7}{15(8!)} + \cdots\right) \quad (55)$$

Many simple and useful relations can be derived from the following, some of which have been published by Mr. Hughes (48) (49):

$$s^2{}_2\,\phi_1 = s^2{}_1\,\phi_2 \quad (56a)$$

$$R^2{}_1\,\phi_1 = R^2{}_2\,\phi_2 \quad (56b)$$

$$R_1\,s_1 = R_2\,s_2 = \text{constant} \quad (56c)$$

Typical formulas for s are:

$$s = x\left(1 + \frac{\phi^2}{10} + \frac{29\,\phi^4}{5{,}400} + \frac{127\,\phi^6}{702{,}000} + \cdots\right) \quad (57a)$$

$$s \approx x_1 + \frac{x^5{}_1}{40\,R^2{}_1\,s^2{}_1} \pm \quad (57b)$$

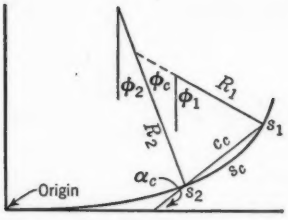

Fig. 8

and

$$s = c\left(1 + \frac{2}{45}\,\phi^2 + \frac{2\,\phi^4}{1{,}575}\cdots\right) \quad (57c)$$

Eq. 57*b* is derived from the cubic parabola. Referring to Fig. 8,

$$s_c = s_1 - s_2 = 2\,R_c\,\phi_c \quad (58)$$

in which s_c is any part of the spiral arc, R_c is its mean radius, and ϕ_c is the subtended angle. Also:

$$R_c = \frac{R_1\,R_2}{R_1 + R_2} \quad (59a)$$

$$\phi_c = \phi_1 - \phi_2 \quad (59b)$$

and

$$s_m = \frac{s_1 + s_2}{2} = 4\,R_c\,\phi_m \quad (60)$$

Some useful approximations can be derived from Eqs. 58 to 60:

$$c_c \approx 4\,R_c \sin\frac{\phi_c}{2} \quad (61a)$$

$$\alpha_c \approx \frac{\phi_1 + \sqrt{\phi_1\,\phi_2} + \phi_2}{3} = \frac{\phi_1 + 4\,\phi_m + \phi_2}{6} \quad (61b)$$

and

$$c_c = s_c - \text{correction} \quad (61c)$$

The correction in Eq. 61*c* is nearly proportional to $(\phi_c)^2$, which fact makes it possible to traverse a spiral along its chords.

By Eq. 50 the first chord correction (see Table 2) is:

$$\text{Correction} = -\,s \times \frac{2\,(\phi^\circ)^2}{45} \times \frac{\pi^2}{180^2} \quad (62)$$

That is,

$$-\,s \times \frac{2\,(\phi^\circ)^2}{45} \times \frac{\pi^2}{180^2} = -\,50 \times 0.5^2 \times 0.0000135386 = -\,0.00016923 \text{ ft.}$$

Assume the following conditions as an example: $s_1 = 300$ ft, $\phi_1 = 18°$, $R_1 = 477.465$, $x_1 = 297.053$ ft, $y_1 = 31.195$ ft, $\tau = 149.508$ ft (by Eq. 54*b*,

$r = 149.509$ ft), $\sin_s \phi = 0.9901754$, and $\text{vers}_s \phi = 0.1039838$. Referring to Table 2, x_1 is computed as 297.050 ft, which checks the foregoing value of 297.053. Similarly, 31.195 checks the foregoing value of y. The values of $\phi°$ in Col. 2, Table 2, are proportional to (point)^2 in Col. 7; thus, for point 5, $\phi° = \frac{25}{36} \times 18° = 12.5°$, etc. In Cols. 8 and 9, a is the difference between adjacent values of (point)^2, the values of a^2 being factors used in determining the chord correction in Col. 10. Eq. 61*b* is used for computing the values of $\alpha°_c$ in Col. 4. The length of any given chord is equal to s_c minus the correction for that chord computed by Eq. 62 and Col. 9.

TABLE 2.—COMPUTATIONS FOR TRAVERSING ALONG SPIRAL CHORDS

Point	Station	$\phi°$	Chord (ft)	$\alpha_c°$	$\cos \alpha_c$	$\sin \alpha_c$	(Point)^2	a	a^2	Correction (ft)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	0 + 00	0.0					0			
			50.000	0.167	0.999996	0.002909		1	1	−0.00016923
1	0 + 50	0.5					1			
			49.998	1.167	0.999793	0.020361		3	9	−0.002
2	1 + 00	2.0					4			
			49.996	3.167	0.998473	0.055241		5	25	−0.004
3	1 + 50	4.5					9			
			49.992	6.167	0.994214	0.107421		7	49	−0.008
4	2 + 00	8.0					16			
			49.986	10.167	0.984298	0.176512		9	81	−0.014
5	2 + 50	12.5					25			
			49.980	15.167	0.965169	0.261628		11	121	−0.020
6	3 + 00	18.0					36			

Many trick problems can be solved directly. For example, given s and σ, let $z = \frac{12\,\sigma}{s}$. Then

$$\phi = z + \frac{z^3}{28} + \frac{397\,z^5}{129{,}360} + \frac{55{,}437\,z^7}{162{,}993{,}600} + \cdots\cdots\cdots\cdots (63a)$$

Another case arises when a given curve center and a given spiral s are fixed. Let R equal the radius of the existing circle; it is required to compute the new radius R_n. If $q = \frac{s}{2\,R}$—

$$\phi = q + \frac{q^3}{6} + \frac{25\,q^5}{504} + \frac{20{,}167\,q^7}{1{,}164{,}240} + \cdots\cdots\cdots\cdots\cdots (63b)$$

from which, knowing ϕ and s, $R_n = \frac{s}{2\,\phi}$ can be computed readily.

As indicated by the authors, the spiral and its involute wind about a point (35) (47) (49) that lies on a 45° line from the origin and is distant from it a length $\sqrt{\frac{\pi\,R\,s}{2}}$. This is the value of c as ϕ approaches ∞. The maximum chord and maximum ϕ-values have been calculated. The involute of Euler's spiral is not the hyperbolic spiral; nor is the evolute of a hyperbolic spiral a simple transition curve.

The idea of a curve that is transitional throughout has occurred to many engineers and, as indicated in the paper, it presents a grade transition problem where the curves meet. Compound spirals similar to compound circles are feasible and the writer has seen this arrangement on a set of plans only once.

The authors are to be commended for their extensive research as indicated by the bibliography. Their list of "approximate relations" will be found useful and may be verified by plotting corrections to s on either log-log paper or alinement graphs.

It should be obvious from the foregoing that Euler's spiral is an exact curve and that tables can be used for most of the work. Some writers use the degree-of-curve system. This should be a personnel choice because either system works if one can establish a definition for it. Since s, R, and ϕ (or Δ) are quantities used by all surveyors, there seems to be little need for any particular one of a dozen degree systems.

Right of ways can be described as parallel to a center-line spiral and this center-line curve can be retraced from any two parts such as s and R or R and ϕ. Areas can be computed by Eq. 52, and they can be checked approximately by the traverse.

The use of tables has been emphasized because the writer has seen many men shrink from the extensive formulas and derivations. The body of data presented herein is only a small portion of the material available and it should be needed only on rare occasions. Eqs. 51 to 63 should prove useful particularly to any designer trying to aline a transition around a bridge pier. The subject of computing a spiral between two circles has not been given much attention in this discussion. Most cases are quite simple if Eqs. 56 are combined with Eqs. 58, 59, and 60. These are exact equations and will not require extensive tables.

RENÉ S. PULIDO Y MORALES,[9] ESQ.[9a]—The use of any of the easement curves to provide a safe and comfortable transition around a curve is only a rough method of obtaining the maximum possible high speed with safety and comfort. It is necessary to study more intently the path and characteristics of the center of gravity of the vehicle, endeavoring to keep it approximately at the same height, and on a predetermined trajectory. Another cause of discomfort is the acceleration of the center of gravity of the vehicle in a transverse direction. This effect cannot be eliminated, but it can be reduced.

The spiral that is transitional throughout is composed of two symmetrical spirals of a length:

$$S = R_m \Delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (64)$$

in which Δ, the total deflection of the curve, is in radians, and R_m is the radius at the point of maximum curvature. Therefore, the total length is equal to:

$$S_t = 2 R_m \Delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (65)$$

Also,

$$S = 1.6 \frac{V^3}{R_m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (66)$$

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[9a] Received by the Secretary August 23, 1945.

Combining Eqs. 64 and 66, and solving for R_m:

$$R_m = \sqrt{\frac{1.6\ V^3}{\Delta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67)$$

Eq. 67 is a valuable guide for determining the minimum radius of a spiral that is transitional throughout.

A mathematical expression whose derivation is readily available in standard texts is

$$\gamma = -\frac{V^3}{R^2}\frac{dR}{dS} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (68)$$

Eq. 68 demonstrates that the spiral is the most advantageous of all types of transition curves. It yields the spiral of minimum length and is particularly useful when space is restricted.

Integrating Eq. 68:

$$S = \frac{V^3}{R\,\gamma} + C. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (69)$$

Noting that $\frac{V^3}{\gamma} = K$ is a constant:

$$S = \frac{K}{R} + C \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (70)$$

which is the typical spiral equation. The formula for the determination of the transition length—$S = \frac{V^3}{R\,\gamma}$ (Eq. 16*b*)—is not precisely applicable to other transitions than the spiral. It is derived by the following reasoning: In the path of a vehicle traveling on a transition curve at constant speed, the centripetal acceleration is zero at the beginning where the radius is equal to ∞, and $\frac{v^2}{R}$ at the end where the radius is the minimum for that case. If t is the time of travel, the rate of acceleration along the curve is expressed by:

$$\frac{v^2}{R} \times \frac{1}{t} = \gamma \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (71)$$

Substituting $t = \frac{S}{v}$ in Eq. 71, $S = \frac{v^3}{R\,\gamma}$; but this is only an expression for the mean value and cannot be accepted as an elegant conclusion, although it is admitted that mean acceleration is equal to maximum acceleration.

The mathematical requirement for the perfect easement curve—that the radius of curvature r at any point should be inversely proportional to the distance s to that point—is not precise, since these two quantities vary approximately (but not precisely), in inverse ratio, as stated in the paper.

In the writer's opinion, all the easement curves—the cubic parabola, the lemniscate, the Talbot, Searles, Crandall, and Ericson curves—are satisfactory and give substantially the same results in the field. The lemniscate advocated by F. G. Royal-Dawson (24) in England is the most rational at the single point where the radius of spiral curvature is greatest; but it is not rational for

any other point along the spiral. This easement curve requires more length than the spiral.

The paper is a good contribution to the literature on easement curves.

C. C. WILEY,[10] M. AM. SOC. C. E.[10a]—Every one who uses the spiral must, at some time, learn how. Consequently, in addition to being mathematically sound and practical in application, the treatment should be such that it can be comprehended easily. The first requirement is that the basic geometry shall be presented simply and clearly, and that it be kept constantly in the foreground The second is that formulas, computations, and operations shall be simple, direct, universally applicable without auxiliaries, and conform to accepted engineering practices.

Most writers on spiral transitions have failed in these things—partly, because they seem to have been too anxious to plunge into mathematical derivations, but more particularly, because they have attacked the problem in such a manner, and from such premises, as to result in cumbersome formulas, tedious computations, and awkward field work, which have obscured the geometry and given the spiral an unmerited reputation of complexity.

The purpose of a curve is to provide a safe, convenient, and comfortable change of direction. The critical factor in traveling on curves is the centrifugal force, which must be kept within satisfactory limits. Basically, centrifugal force is a function of the angular velocity which, in turn, is a function of curvature. The conventional formulas for centrifugal force, however, have been written in terms of linear velocity and radius of curvature. This may explain why so many writers have started with the radius of curvature instead of curvature itself.

The circle is universally used as the basic curve because its geometry is simple; it is symmetrical with respect to traffic; and conditions of travel on it are constant. Its defect lies in that, mathematically, it requires instant change from rectilinear to curvilinear motion, which is mechanically impossible. Consequently, spirals are introduced at the ends of the circular arc to provide a gradual transition in curvature to control the centrifugal force. Any change of direction made by the spiral is only a by-product.

A circle may be defined properly as "a curve whose angular change of direction per unit of arc is a constant." When expressed in ordinary engineering units this is nothing more nor less than the familiar "degree of curve" whose merits have been recognized by railroad engineers for about a century, but which many highway engineers still ignore. The degree of curve expresses the curvature of the circle directly, whereas the radius is merely an incidental linear dimension useful in certain computations. Since the spiral is designed to provide transition in curvature, it would seem that the "rational" attack would be on the basis of curvature rather than radius.

All writers on this subject seem to agree that the ideal spiral should be a curve whose curvature increases uniformly with the distance from the origin. Therefore, the intrinsic equation of the spiral may be written directly from the

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[10a] Received by the Secretary September 26, 1945.

definition in the form

$$d = k\,s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (72)$$

which, in engineering terms, makes d the degree of curve at any point; k the rate of change of the degree of curve in degrees per 100 ft (station); and s the distance from the origin in stations (100 ft). Every term now has a simple geometric conception which is easily comprehended and easily learned. An examination of the writings of the late A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and those of others using his method shows that it has many intrinsic advantages.

In practice, the curvature of the basic circle is selected with respect to expected speed subject to topographic restrictions. A length of arc is then cut from it to provide the required total change of direction. Rarely, if ever, is the length chosen and the curvature made to conform. In contrast, most writers have chosen the length of spiral and made the curvature to fit; probably because the first spirals were to be superimposed on existing arbitrary lengths of superelevation runoff. This mode of attack, especially when combined with the radius of curvature, inevitably leads to cumbersome formulas, complicated computations, and other troubles, which have obscured, and sometimes almost obliterated, the basic geometry. As stated, the basic geometry must be understood if the spiral is to be used intelligently.

Under the Talbot method, when k is chosen, a spiral of indefinite length is "fixed" from which arcs of proper length may be cut to fit between a tangent and any circle, or between two circles as in the case of the compound curve. Thus, this method becomes fully "rational" and of simple and universal application.

Various writers have shown that the curvature characteristics of a satisfactory spiral are a function of the cube of the speed. The authors express this in Eq. 16*b* which is not readily usable, whereas R. A. Moyer, Assoc. M. Am. Soc. C. E. (19) gives it in the simple arithmetical form

$$L = \frac{1.58\,V^3}{R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

stating that this is the minimum length (L) for safety. In order to provide for both safety and comfort, the writer and the late G. W. Pickels, M. Am. Soc. C. E., have recommended a desirable highway spiral in the form

$$k = \frac{230{,}000}{V^3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

which is practically equivalent to a coefficient of 2.5 in Eq. 73 and gives a spiral one half as long as that considered desirable on the railroads (21).

Since moderate variations of the speed are permissible on any curve, and since the same speeds will apply over moderate variations in curvature, it follows that a comparatively small number of different spirals having simple and convenient values of k will fully cover all ranges of speed and curvature to be found in highway alinement. On the other hand, the use of arbitrary

lengths, each covering a range of curvature, results in inconvenient values and requires the computation of an entirely new spiral for each change of curvature. Thus the Talbot method of fixing the spiral curvature and selecting a length to meet the given circle provides simple formulas, a minimum of computations, and a maximum flexibility in application.

The term ϕ, total change of direction at any point on the spiral, appears in many of the authors' formulas, but none of the formulas seem to give an expression for evaluating it. Perhaps this is a result of the idea of making $\phi = \frac{1}{2}\Delta$; but if Eq. 4 is extended a step or two further, the result is:

$$\phi = \tfrac{1}{2} k s^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

which is identical in form with the Talbot formula but is in different units. In the Talbot formula ϕ is given directly in degrees, whereas the authors give it in radians, which is undesirable.

Various writers show that α, the spiral deflection angle, is very nearly $\frac{1}{3}\phi$ and, in Table 1, the authors give corrections from which they show that in most cases the difference is very small, and may be omitted. Professor Talbot came to the same conclusion. As a matter of fact, in practice α can always be taken as $\frac{1}{3}\phi$ because the sole effect of omitting the correction is to move the circular curve bodily a very small distance outward. In a very extreme case, with $\phi = 30°$, this movement was only about 1 in., which is utterly insignificant.

In Eq. 75, α becomes $\frac{1}{6} k s^2$ in degrees which, when converted to minutes, becomes the ridiculously simple equation

$$\phi = 10 k s^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

Obviously, with uniform chord lengths, it is necessary to compute only the first value of α by the formula and then find the others by multiplying the first by the square of the digits. If simple values of k are used with stakes at the usual distances of 25 ft, 50 ft, or 100 ft, the spiral deflections may be written almost as fast as those of a circular curve (with possibly a last one equal to $\frac{1}{3}\phi$ if the spiral has an odd length). This is so simple, and so in accord with basic geometry, that it is easily learned.

The general solution for the tangent distance of a spiraled curve, in Section 8, is interesting. It is cumbersome because it involves the calculation of the otherwise useless spiral chord c and the solution of one right triangle and two oblique triangles. The geometry of the solution is well shown in the figure, but the authors "spoiled the show" by writing out all the mechanical operations, culminating in the appalling appearance of Eq. 18, which completely obliterates the geometry. The day is past when either engineers or students want, or need, a formula like this which only adds to the forbidding appearance of the spiral treatment.

A few additions to Fig. 3 would have shown directly that

$$T = (R + \sigma) \tan \tfrac{1}{2}\Delta + \tau \ldots\ldots\ldots\ldots\ldots\ldots (77)$$

which is far simpler as it involves the solution of only one right triangle plus two spiral elements often wanted for other purposes. This solution is shown

by the authors in approximation 7, Section 11, in which the only approximation is the very slight one of using $\tau = \frac{1}{2} S$.

Obviously, safety and comfort are highly important but it must not be inferred that they have not received, or are not receiving, careful attention. The railroads originally derived lengths of runoff from studies of safety and comfort, and later derived their spirals in a similar manner. Highway engineers, however, had little factual data to aid them until the work of Professor Moyer appeared (19). Unfortunately, present data are scarcely adequate and those that are available must be used with care and judgment.

Eq. 15 may be reduced to the slightly approximate "safe speed" formula often used by highway engineers

$$V^2 = 15\,R\,(e + f) \qquad (78)$$

in which: e is the actual superelevation cross slope built into the road; and f is usually defined as the safe coefficient of friction against lateral skidding. Actually, it is a coefficient of unbalanced centrifugal force expressed in terms of cross slope. Thus, the quantity $e + f$ becomes the equivalent theoretical superelevation for the maximum speed V. If the maximum value of f against skidding only is used, no allowance is made for comfort; hence the latter must be introduced merely by judgment in choosing f since, at present, factual data are not available.

Shortly after the appearance of Professor Moyer's work (19), the U. S. Public Roads Administration suggested a value of $f = 0.16$. Several highway departments objected and several made test runs. The Illinois Division of Highways, on the basis of both safety and comfort, reached the conclusion that a maximum value of $f = 0.10$, for speeds of approximately 50 miles per hr, was satisfactory. An examination of available data indicates that, for both safety and comfort, f may be taken as about 0.10 at 40 miles per hr and about 0.04 at 100 miles per hr. This agrees well with the railroad rule for excess speed which is equivalent to about $f = 0.05$.

The idea of curves "transitional throughout" is an old one. It seems to result from a misinterpretation of "transitional" and the failure to give due consideration to the elements of comfort and convenience on curves. The primary function of the spiral is to make a transition in curvature to control centrifugal force—not to provide change of direction. To require also that it provide change of direction is to impose a duty for which it is not fitted.

When traveling on a tangent both driver and passengers are subject to constant conditions. In traveling on a circle they are also subjected to constant conditions although these conditions are not the same as on a tangent. The function of the spiral is to provide a transition between these different conditions. With proper spirals the change is made with comfort and convenience as well as safety, and the driver and passengers must adjust themselves to changing conditions only on the spirals. If the curve is "transitional throughout" there is no part of its entire length where travel conditions are constant.

Furthermore, since a spiral is twice as long as the circle with the same final curvature and total central angle, making the curve transitional throughout adds greatly to the length of curve over all of which travel conditions are con-

stantly changing. For example, assume a speed of 60 miles per hr on a 4° curve ($R = 1{,}432.5$) with $\Delta = 64°$. A spiral with $k = 1$ is suitable, giving spirals 400 ft long. The unspiraled circle would have a length of 1,600 ft, with bad conditions at the end; but most of its length would offer constant travel conditions. When spiraled, the total length would be 2,000 ft, of which 1,200 would have uniform travel conditions with 400 ft at each end to make the transitions. If "transitional throughout," the length would be 3,200 ft with no part having constant conditions. If the latter curve is reduced to 2,000 ft its central curvature would be increased to 6° 24′ ($R = 895.3$), which is entirely too sharp for the given speed.

If, for a given speed, curvature, and change of direction, it should happen that $\phi = \frac{1}{2}\Delta$, the curve is short and automatically and properly is transitional throughout. If the computed value of ϕ is greater than $\frac{1}{2}\Delta$ then, because of speed, the same spiral should be used but it should merely be shortened until $\phi = \frac{1}{2}\Delta$. If the computed value of ϕ is somewhat less than $\frac{1}{2}\Delta$, speed would again call for the same spiral and the residual circular arc, no matter how short, should be retained for simplicity. Making $\phi = \frac{1}{2}\Delta$ would not improve the curve and would require completely new calculations and field notes. For greater values of Δ the benefits of uniform travel conditions on the curve would become increasingly evident.

In 1923 the writer had a graduate student who spent much time in trying to develop a curve "transitional throughout." Several such curves were laid out and test runs made over them. Except for short curves they were disliked by both drivers and passengers. If they were made of suitable middle curvature, the lengths became unreasonably long, and if they were shortened to reasonable length, the middle curvature became too sharp for the speeds. In addition, the geometry was complex, the formulas ponderous, the calculations long and tedious, and the field work awkward. In short, the results were so unsatisfactory that they were never published.

The spiral is intrinsically so simple that it is a misfortune that it should have been so mutilated as to be almost unrecognizable, and to acquire a forbidding if not sinister appearance. When properly presented, its applications are so simple that an average field engineer can make complete computations and field notes in very little more time than is required for an unspiraled curve and the field operations are practically identical. In some schools students are required to be able to spiral, completely, either a simple or a compound curve with no auxiliaries whatever (not even the textbook) except the usual tables of the trigonometric functions. Until something simpler than this is devised the writer is perfectly contented with his present methods.

To summarize the writer's honest view of this paper: It contains nothing really new in mathematical treatment. The general mode of attack is the same as that used by a number of other writers, as are many of the forms of mathematical derivations. It adds nothing to expedite or improve the use and applications of the spiral; if anything, it confuses them more than before. The use of ϕ in the formulas is an innovation, but it is objectionable for several reasons. The principal objection is that it is based on a false premise since ϕ, the total change of direction on the spiral, is an incidental result and not a

basic function. Some of the incidental items such as "safety and comfort" have merit but might have been given more attention. The "approximations" have all been long recognized whereas the manner in which they are given still leaves a hazy impression that they should not be tolerated.

What the highway spiral needs is some way of eradicating all the old awkward treatments of the spirals as given in most field manuals, and of avoiding the need for tabular data. State highway departments are wasting plenty of persons, pencils, paper, patience, and profanity on a problem which, actually, is little, if any, more complex than circular curves.

Corrections for *Transactions:* In May, 1945, *Proceedings*, on page 660, line 9, change "Setting" to "Solving for α and setting" and line 23, change "added to" to "subtracted from"; on page 663, line 2, change "0.4364" to "0.1091" and "25°" to "6.25°" and line 3, change "50°" to "12.5°"; on page 665, line 17, change "varies approximately as $0.5\,x$, and" to "is approximately equal to $0.5\,x$," line 18, change "therefore" to "that is," line 30, change "$\frac{1}{6}$" to "$\frac{1}{3}$," and, in Eq. 23, change the denominator from "6" to "3"; and, on page 666, line 2, change "of" to "up to."

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DISCUSSIONS

ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES USING REDUCED EQUATIONS

Discussion

BY LEE H. JOHNSON, JR.

LEE H. JOHNSON, JR.,[16] M. AM. SOC. C. E.[16a]—The writer is indebted to those who have prepared discussions of this paper. He has been aware of the fact that it presented only an idea, and was prompted to submit it in the hope that others would be stimulated to investigate the possibilities of the idea. In statically indeterminate structures both the geometry and the properties of the cross sections and materials have an influence upon the reactions and the internal forces. It is conceivable that the relation between these two influences may be determined sufficiently to be of value to the designer, at least for some types of structures. The use of reduced equations may provide a method of limited applicability for determining this relation. The possibilities of the method for this purpose were illustrated in the application to two-hinged arches, where it was shown that the influence of cross sections and materials upon reactions was relatively small and could be correlated to the influence of geometry.

Mr. Oesterblom's discussion is a great encouragement to pursue the idea still further. The writer agrees with him that at present the possibilities of the method largely remain to be determined. The writer is indebted to Mr. Polivka for emphasizing the usefulness of the two-hinged arch data in eliminating certain cumbersome computations.

Although Mr. Beskin's mathematical analyses appear to be correct, his criticism comes from misleading inferences which he draws from them. He points out several apparent limitations to the writer's method—one to the basic principle as expressed in Eq. 1, another to the computation of moment values, M_0, in Eq. 36, and a third in the computation of thrust for a variable load in Eq. 45. None of these are actual limitations since they are valid only for values in the neighborhood of zero.

NOTE.—This paper by Lee H. Johnson, Jr., was published in November, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1945, by Josef Sorkin; May, 1945, by I. Oesterblom, and Leon Beskin; and June, 1945, by Jaroslav Polivka.

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[16a] Received by the Secretary October 22, 1945.

Consider two influence lines whose corresponding ordinates differ by relatively small amounts and which do not cross the horizontal or zero axis at the same point. It is obvious that the relative error becomes infinite at one of the zero points and is very large in the vicinity of that point. The small ordinates to influence lines, moment diagrams, and shear diagrams near the zero points are seldom significant in a design. Mr. Beskin has attempted to make them so, in discussing the writer's two-hinged arch data, by using a combination loading consisting of a uniform load and a load that increases parabolically from the center to the ends of the arch. He finds under this loading that the error in thrust, using the reduced equations, is infinite for a certain ratio of the two load constants, which ratio, it should be noted, is negative. Again it is obvious that if an arch is subjected to two loads, one downward and one upward as is indicated by the negative sign, the thrust on the arch can be made zero by properly balancing the loads, in which case the error in making any approximation to the thrust must be infinite. The stresses computed with a thrust of zero and the stresses computed with a very small thrust would be approximately the same in spite of the fact that the error in thrust is infinite. In a problem of this type where the thrust is zero, or is so close to zero that the approximation may be of the opposite sign, the thrust is not significant in determining the stresses and the arch behaves more as a curved beam with unusual support and loading than as an arch. Contrary to Mr. Beskin's statement, the method of reduced equations can be used to examine loadings of this type, and he, himself, has made a start in this direction.

The comparative values which Mr. Beskin shows in Table 8 are with respect to the basic influence values rather than with respect to curve A or B, Fig. 12; hence the large relative errors. He contends that there is no justification for curves A and B; but the writer is of the opinion that much more justification for these curves can be found in mathematical analyses than can be found for the use of any "allowable stress" in the design of a structure.

The writer made no claim to a difference between parabolic and circular arches such as Mr. Beskin states. He did claim, and still does, that the "basic influence values" for circular arches are not a linear function of the (L/h)-ratio as are those of the parabolic and semi-elliptic arches. The term "basic arch" was not used in the paper.

Mr. Beskin's suggestions about the terminology are noteworthy. The writer agrees that the term "correlation" would have been better than "correspondence" and that "error" would have done as well as "difference term." He prefers the term "reduced equation," however, as it implies a particular type of approximation. There is a reason to assume that the correspondence in Fig. 1(*c*) is stopped precisely at the center line, for it is precisely there that the slope of curve F changes from positive to negative.

Mr. Sorkin's criticism arises from a misinterpretation of the details of the paper, the significance of the fundamental idea, and of its application to two-hinged arches. Apparently, he denies the possibility of beginning the design of a statically indeterminate structure without first assuming the size and properties of the members. The writer has shown mathematically under what conditions these quantities may be omitted from the first computations.

Mr. Sorkin states that the method is too cumbersome for practical use in the case of steel ribs composed of plates and shapes because of the involved mathematics. The reader should understand that the result of the application of the method to two-hinged arches is a set of three charts, Figs. 5, 10, and 12, which apply to steel girder ribs and to truss ribs as well as to solid concrete ribs. In the paper the writer did not give the tabular data for plotting the basic influence line, curve A, and curve B in Fig. 5. These data are given in Table 9.

TABLE 9.—ORDINATES FOR INFLUENCE LINES FOR PARABOLA
(SEE FIG. 5)
(Values Are in Terms of L/h)

Fraction of span	Basic influence line	Curve A	Curve B
0.05	0.028	0.031	0.036
0.10	0.058	0.061	0.069
0.15	0.085	0.090	0.100
0.20	0.112	0.116	0.124
0.25	0.136	0.139	0.144
0.30	0.159	0.159	0.159
0.35	0.177	0.175	0.169
0.40	0.192	0.186	0.176
0.45	0.201	0.193	0.179
0.50	0.204	0.195	0.180

The parabolic formula, Eq. 32, which Mr. Sorkin mentions, can be used with small errors only for parabolic arches, and for circular arches of the higher (L/h)-ratios—say greater than 6. It will not give accurate results for these arches in cases where the crown thickness is much greater than the thickness at the haunch. The writer's data give closer approximations in all cases than the parabolic formula and also apply to any other arch form within the limits of the data in Figs. 10 and 12. The writer does not agree with Mr. Sorkin's implication that since "the difference between one or two, or even three, trial designs reflects the total effort only in a minor way," no attempt should be made to improve this phase of the work in the creation of a structure.

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DISCUSSIONS

FLOW IN A CHANNEL OF DEFINITE ROUGHNESS

Discussion

BY C. J. POSEY

C. J. POSEY,[40] M. AM. SOC. C. E.[40a]—Engineering uses of the science of hydraulics have about reached the point where improvement of the favorite open-channel friction formulas is economically justified. Professor Powell's paper should help to convince the complacent that research on this fundamental problem is badly needed. Although the Kutter coefficient can be predicted to within a thousandth or two, under favorable circumstances, the corresponding uncertainty in the major variables is appreciable. Where long channels form important links in great projects, this lack of precision can be costly.

The two important variables that give trouble, in evaluating friction flow in open channels, are the roughness and the shape of the cross section. Thorough investigation of the effect of the latter has been avoided, in obtaining results for practical use, by testing shapes similar to those used in design. The effect of channel shape has been oversimplified in the "rational" formulas for open-channel flow. A reliable means of evaluating the effect of channel shape, well substantiated by experimentation, would seem to be a necessary part of any satisfactory solution of the roughness problem. It would also free the designer from adherence to traditional shapes, and enable him to take full advantage of new developments in construction methods.

NOTE.—This paper by Ralph W. Powell was published in December, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1945, by Joe W. Johnson and E. A. LeRoux; and October, 1945, by Garbis H. Keulegan.

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